ON THE BEHAVIOR OF SOLUTIONS OF THE CROSS DIFFUSION SYSTEM

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In this paper, we studied the qualitative properties of solutions of a nonlinear cross-diffusion system coupled via nonlinear boundary conditions

$$\begin{cases} \dfrac{\partial u}{\partial t}=\dfrac{\partial}{\partial x}\left(\upsilon^{m_1-1}\left|\dfrac{\partial u^k}{\partial x}\right|^{p-2}\dfrac{\partial u}{\partial x}\right), \quad x\in R_+,\ t>0, \\ \dfrac{\partial \upsilon}{\partial t}=\dfrac{\partial}{\partial x}\left(u^{m_2-1}\left|\dfrac{\partial \upsilon^k}{\partial x}\right|^{p-2}\dfrac{\partial \upsilon}{\partial x}\right), \quad x\in R_+,\ t>0, \end{cases} \tag{1}$$

$$\begin{cases} -\upsilon^{m_1-1}\left|\dfrac{\partial u^k}{\partial x}\right|^{p-2}\dfrac{\partial u}{\partial x}(0,t)=u^{q_1}(0,t), \quad t>0, \\ -u^{m_2-1}\left|\dfrac{\partial \upsilon^k}{\partial x}\right|^{p-2}\dfrac{\partial \upsilon}{\partial x}(0,t)=\upsilon^{q_2}(0,t), \quad t>0, \end{cases} \tag{2}$$

$$u(x,0)=u_0(x), \quad \upsilon(x,0)=\upsilon_0(x), \quad x\in R_+, \tag{3}$$

where $k,\ p,\ m_1,\ m_2>0,\ q_1,\ q_2>0,\ u_0,\ \upsilon_0$ are nonnegative continuous functions with compact supports in R_+.

Cross-diffusion patterns are found in various spheres of natural sciences. For example, in physical systems (plasma physics) [1, 3], in chemical systems (dynamics of electrolytic solutions), in biological systems (cross-diffusion transportation, dynamics of population systems), in ecology (dynamics of the forest age structure), in seismology – the Burridge-Knopoff model describing the interaction of tectonic plates. Mathematical models with cross diffusion are widely used in the study of biological population and tectonic plate movement [3].

The study of the conditions of global solvability and insolubility of problem (1) - (3) for various values of numerical parameters has been the subject of a large number of papers [2-6] (for details see the reference [3, 6]).

This paper is devoted to the study of the asymptotics of a self-similar solution of problem (1) - (3). Various self-similar solutions of problem (1) - (3) are constructed for the case of slow diffusion ($m_1, m_2>1$), which are the asymptotics of the solutions of the problem in question. For a numerical study, the methods are proposed for selecting the appropriate initial approximation for the iterative process, which preserve the qualitative properties of problem (1) - (3). An iterative process was designed and numerical calculations, showing rapid convergence to the exact solution, were performed.

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