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MATHEMATICAL MODEL AND NUMERICAL RESEARCH ON MODELING OF THE COIL SPRING OF THE AXLE SUSPENSION OF HIGH-SPEED ELECTRIC TRAINS

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ABSTRACT

The article presents a mathematical model and a developed algorithm for numerical studies on the selection of rational parameters of a modernized coil spring for axle box spring suspension of high-speed electric trains, the design of which is protected by patent of the Republic of Uzbekistan for invention No. IAP 05219 [1].

The foundations of the theory and practical methods for studying the dynamics of vehicles were laid by N.P. Petrov, I.E. Zhukovsky, S.P. Timoshenko, D.H. Young, U. Weaver, I.M. Babakov, and then developed by A.M. Goditsky - Tsvirko, M.V. Vinokurov. In the works of M.F. Verigo, S.V. Vershinsky, S.M. Kutsenko, V.A. Lazaryan, V.B. Medel, I.I. Chelnokov, modern methods for studying natural and forced vibrations of rail vehicles, their interaction with the track superstructure were most fully developed. A significant contribution to the solution of these problems was made by the works of E.P. Blokhin, L.O. Grachev, V.P. Ivanov, I.P. Isaev, A.A. Kamaev, L.N. Nikolsky, E.N. Nikolsky, A.N. Savoskin, V.P. Koturanov, M.M. Sokolov, L.A. Shadur, I.P. Kiselev, E.S. Oganyan, V.I. Kiselev, M.A. Ibragimov [1,2,3] and others.

Research has been conducted and is being conducted on this topic by leading scientists worldwide such as S.A. Brebbia (Wessex Institute of Technology, UK), G.M. Carlomagno (University of Naples di Napoli, Italy), A. Varvani-Farahani (Ryerson University, Canada), S.K. Chakrabarti (USA), S. Hernandez (University of La Coruna, Spain), S.-H. Nishida (Saga University, Japan). Authoritative scientific schools and prominent scientists in the CIS countries from MIIT, PGUPS, MAI, VNIIZhT, JSC VNIKTI, JSC Russian Railways, etc. worked on these issues. A significant contribution to solving many complex problems and checking theoretical conclusions related to the study of the processes of oscillations of the spring suspension of the rolling stock was made by the Russian Research Institute of Railway Transport (CNII MPS) and the Russian Research Institute of Railcar Building (NIIV), where along with theoretical studies, a large number of experimental studies (bench and full-scale ones) were conducted [1,2,3]. In Uzbekistan, the academician of the Academy of Sciences of the Republic of Uzbekistan, professor, doctor of technical sciences Glushchenko A.D., professors Fayzibaev Sh.S., Khromova G.A., Shermukhamedov A.A., Adylova Z.G., Rakhimov R. V., Ruzmetov Ya. O., Khamidov O. R., Radjibaev D. O. and their students studied the problems of optimizing the systems of spring suspension of rolling stock [5÷8].

However, in the existing calculation methods, the curvilinearity of surfaces, impulse contact processes that occur during the operation of the spring suspension of ground vehicles, the complexity of the dynamic loading pattern, and the volumetric configuration of systems have not been taken into account so far.

To derive the equations of spatial oscillations of a cylindrical elastic rod bent along a helix with a variable radius of curvature of the coils, we used the results obtained in [5÷8] and the following assumptions.

1. The boundary element is taken as a single coil of a cylindrical elastic rod bent along a helix with a fixed radius of curvature (Fig. 1). N is the number of boundary elements (depending on the spatial configuration of the spring element) (Fig. 2); it is connected into a single dynamic system using boundary conditions.

2. One coil is described by a curvilinear coordinate system (Fig. 2), characterized by distance ℓ to fix the location of a particular section, measured along the length of a helical line bent along the radius R_s in plane $Y\ \ell$, passing through the centers of gravity of these sections.

The parameters of the sections of the spring coil (Fig. 1) are taken into account according to

- cross-sectional area $F_s = \frac{\pi d_s^4}{4}$;

- mass intensity $M_1 = \frac{\pi d_s^2 \cdot \rho}{4}$ and i_1 mass moment of inertia, where ρ is the density of the spring coil material;

- equatorial $I_x = I_y = \frac{\pi d_s^4}{64}$ and polar $I_0 = I_x + I_y = = \frac{\pi d_s^4}{32}$ moments of inertia of the cross-sectional area of the spring coil,

- modulus of elasticity of the first E and the second G kinds of the coil material.

$P_{DYN}(t)$

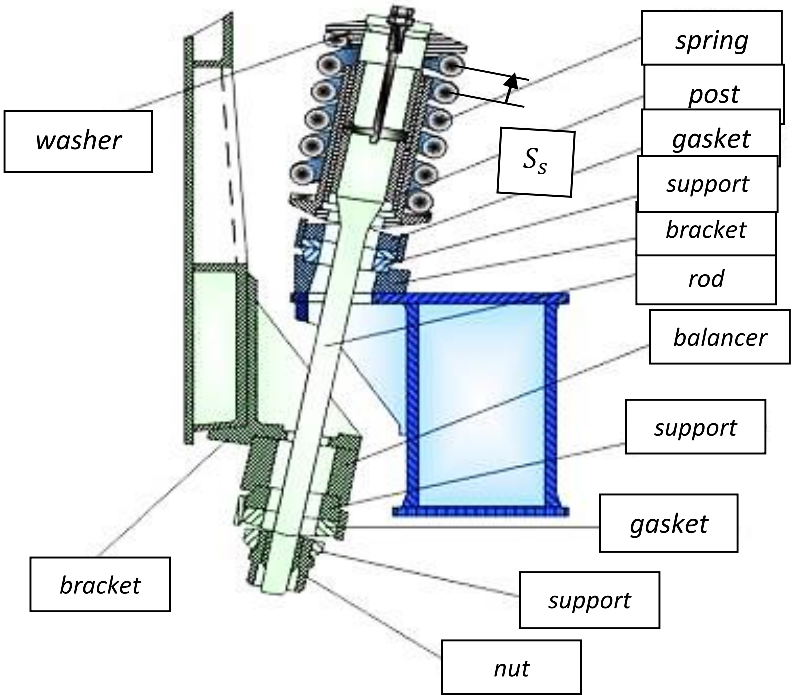

Figure 1. Calculation scheme for simulation of vibrations of cylindrical elastic rod bent along a helix (for swing link of a rail vehicle (a locomotive)).

3. We introduce generalized coordinates that take into account:

- elastic bending deformations $x_s(t,\ell)$, $y_s(t,\ell)$ in two planes - tangent to the helix and parallel to the axis of the cylinder of radius R_s of the winding of the coil, and perpendicular to the first plane;

- elastic deformations under torsion $Q_s(t,\ell)$ and compression $U_s(t,\ell)$ relative to the longitudinal axis of the helix of the spring.

3. With the introduced assumptions, using the Ostrogradsky-Hamilton method [3], we compose the equations of oscillations for one coil of the spring along each generalized coordinate of elastic deformations. Then, using the Euler equation for elastic systems, we obtain, as a result, a system for describing the bending (in two planes), longitudinal and torsional vibrations of a cylindrical rod bent along a helix, which generally characterizes the spatial vibrations of a coil of a helical spring

$$\frac{\partial^4 y_s}{\partial \ell^4} + \frac{\partial^2 y_s}{\partial \ell^2} \cdot \frac{1}{E\,I_y}\left(\frac{2}{R_s^2} + P_s \sin\lambda\right) + \frac{y_s}{R_s^4} + \frac{1}{E\,I_y}\left(M_1 \frac{\partial^2 y_s}{\partial t^2} - i_1 \frac{\partial^4 y_s}{\partial t^2 \partial \ell^2}\right) +$$
$$+\frac{1}{R_s} \cdot \left(\frac{1}{R_s^2}\frac{\partial U_s}{\partial \ell} + \frac{\partial^3 U_s}{\partial \ell^3}\right) = \frac{2\pi \sin\lambda}{\ell_s E\,I_y} \cdot P_s(t)\cos\left(\frac{\ell}{R_s}\right), \qquad (1)$$
$$\frac{\partial^4 x_s}{\partial \ell^4} + \frac{\partial^2 x_s}{\partial \ell^2} \cdot \frac{1}{EI_x}\left(P_s \sin\lambda - \frac{GI_0}{R_s^2}\right) + \frac{1}{E\,I_x}\left(M_1 \frac{\partial^2 x_s}{\partial t^2} - i_1 \frac{\partial^4 x_s}{\partial t^2 \partial \ell^2}\right)$$
$$+\frac{\partial^2 Q_s}{\partial \ell^2} \cdot \frac{GI_0}{R_s E\,I_x} = \frac{2\pi \cos\lambda}{\ell_s E\,I_x} \cdot P_s(t)\cos\left(\frac{\ell}{R_s}\right), \qquad (2)$$
$$\frac{\partial^3 y_s}{\partial \ell^3} + \frac{1}{R_s^2} \cdot \frac{\partial y_s}{\partial \ell} + \frac{F_s {R_s}^2}{I_y} \cdot \frac{\partial^2 U_s}{\partial \ell^2} - \frac{M_1 R_s}{E\,I_y} \cdot \frac{\partial^2 U_s}{\partial t^2} = 0\,, \qquad (3)$$
$$\frac{1}{R_s} \cdot \frac{\partial^2 x_s}{\partial \ell^2} + \frac{\partial^2 Q_s}{\partial \ell^2} - \frac{i_1}{GI_0} \cdot \frac{\partial^2 Q_s}{\partial t^2} = -\frac{1}{GI_0}\cos\lambda \cdot \sin\left(\frac{\ell}{R_s}\right) \cdot P_s(t)\,. \qquad (4)$$

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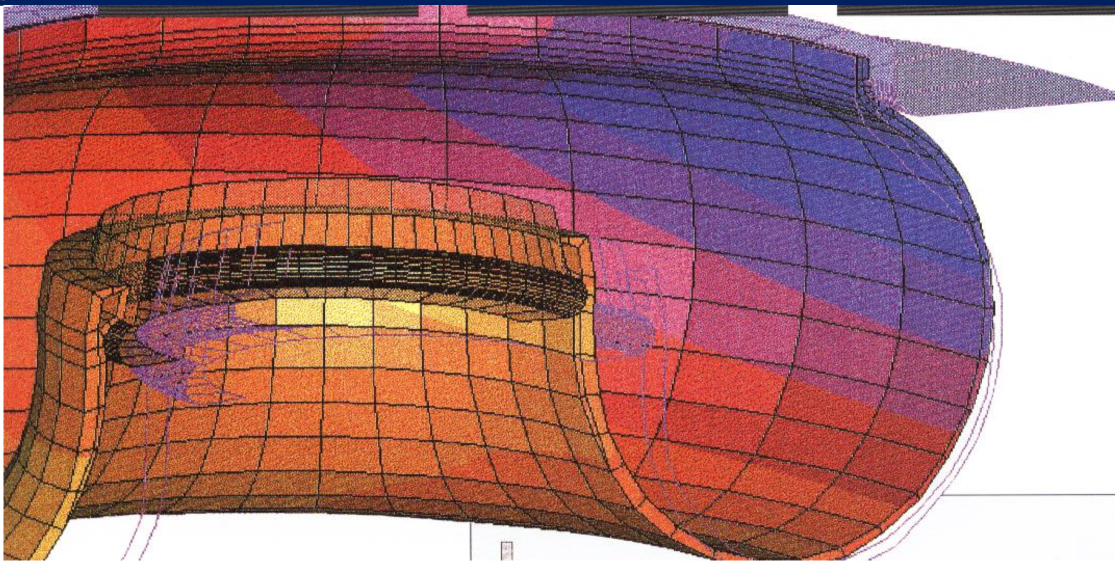

Figure 2. Formation of a complex configuration of a helical spring element by connecting N boundary elements along a helical line with a variable radius of curvature of the turns.

The resulting system of differential equations allows approximate solutions for the cases when $P_s \sin\lambda$ and $P_s \cos\lambda$ are constants. These solutions include functions of static $x_s(\ell), y_s(\ell), U_s(\ell), Q_s(\ell)$ and dynamic $x_a(t,\ell), y_a(t,\ell), U_a(t,\ell)$, $Q_a(t,\ell)$ components.

For numerical calculation in the *MATHCAD 15* programming environment, we accept the following initial data and assumptions:

1. We accept a model of a cylindrical helical spring, characterized by a wire diameter d_s, an average coil diameter D_s, a pitch between turns S_s in an unloaded state, a number of turns i_s, a static load P_s , a lead angle in a loaded state λ.
2. We present the initial model as a single coil with a vertical axis of symmetry and length passing through the center of gravity of the sections of the helix ℓ_s (Fig. 1, a).
3. For the accepted model of a single coil, the upper section for $\ell = 0$ is considered cantilever (free end), loaded with concentrated static load P_s , and the lower section for $\ell = \ell_s$ - is considered clamped.

A method for calculating stresses in a coil spring of a tubular section was proposed as a result of the analytical and numerical studies. The results of the numerical calculation of the stress state of helical springs with swing links for the mainline electric locomotive VL-80s are summarized in Table 1.

Table 1

Selection of rational parameters based on the stress state of a helical spring for spring suspension of the mainline electric locomotive VL-80s (varying the inner diameter d_2)

№№ п/п	Spring vibration parameters	Diameter d_2, in mm			According to [19] for a spring wound from a bar d = 42 mm
		12	16	20	

1. 1.1. 1.2. 1.3 1.4	Eigenfrequencies (5 modes of vibration) p_x p_y p_Q $p_1 \approx$ $\approx p_3$	 1.945 0.989 1.042 5.52 5.05	 2.006 1.03 1.041 5.71 5.24	 2.08 1.09 1.04 5.95 5.47	 - - - - -
2.	Total maximum static stresses σ_{stat} , MPa	636.66	684.7	730.11	546 (not considering the bending)
3.	Total maximum dynamic bending stresses, σ_{DYN}^{bend} , MPa	114.24	154.24	214.84	-
4.	Total maximum dynamic torsion stresses, τ_{DYN} , MPa	-43.97	-94.56	- 145.38	-
5.	Total maximum dynamic stresses, σ_{DYN}, MPa	122.41	180.91	259.66	148.6
6.	Total maximum stresses σ_{max} , MPa	759.07	865.61	989.77	694.6
7.	Tensile strength (endurance limit of spring steels of type 65C228A or 60C2XA), $[\sigma]$, MPa		1000		923
8.	Safety factor $K_{dirability} = \frac{[\sigma]}{\sigma_{max}}$	1.32	1.155	1.01	1.33

According to Table 1, a spring was selected for cradle suspension of the VL-80s main electric locomotive with the following parameters: D=198 mm, H=648 mm, d_1=40 mm, d_2=12 mm, N=6.

The total stresses in the most loaded section of the spring at $\ell = 0.33\,\ell_s$ are in our calculation:

- static at $P_{stat} = 53.50$ kH - $\sigma_{stat} = 636.66$ MPa;
- dynamic (under bending) for $P_{aDYN} = 1.4$ kH - $\sigma_{DYN}^{bend} = 114.24$ MPa;
- dynamic (under torsion) $P_Q = 1.0$ kH - τ_{DYN} = -43.97 MPa;
- total dynamic stresses $\sigma_{DYN} = 122.41$ MPa;
- total maximum stresses $\sigma_{max} = 759.07$ MPa;
- safety factor (for $[\sigma] = 1000$ MPa) $K_{dirability} = \frac{[\sigma]}{\sigma_{max}}$ =1.32.

Conclusions:

1. The proposed numerical-analytical applied method and refined methods of dynamic strength calculation (using the method of boundary elements - Boundary Element Technology) for curvilinear elements of the rolling stock of railways of a complex profile (springs, vibration

dampers, cradle suspension units, spring suspension) are planned to be used in the design , operation and modernization with the extension of the useful life of locomotives.

2. The proposed methods are relevant for the Republic of Uzbekistan, and for the CIS countries, as they allow us to obtain better dynamic characteristics of ground vehicles, which determine their reliability and key performance indices.

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