IKKINCHI TIP MATRITSAVIY SHARNING AVTOMORFIZMI

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Anotatsiya: Ushbu tezisda II - tip klassik sohalarda matritsaviy sharlarning avtomorfizmi o'rganildi.

Kalit so'zlar: klassik soha, matritsaviy shar, matritsaviy sharning avtomorfizmi, shilov chegarasi.

Ko'p kompleks o'zgaruvchili funksiyalar nazariyasi yoki ko'p o'zgaruvchili komleks analiz hozirgi kunda jadallik bilan rivojlanmoqda. Shu bilan birgalikda klassik kompleks analizning bir qancha muammolari hali ham o'z yechimini topgani yo'q. Matritsalar ko'p ozgaruvchili kompleks analiz masalalarini o'rganishda keng qo'llaniladi.

1935-yilda E. Kartan o'zining [3] ishida bir jinsli, simmetrik, qavariq, chegaralangan, to'la doiraviy sohalarni 6 ta tipi mavjud ekanini keltirgan. Shu tiplardan to'rttasiga tegishli bo'lgan sohalar klassik sohalar deyiladi, chunki ularning avtomorfizmlari yarim oddiy Li gruppalaridir. Bu to'rt K_1, K_2, K_3 va K_4 tip quyidagi ko'rinishga ega:

$$K_1 = \left\{ Z \in \mathbb{C}\left[m \times k\right] : I^{(m)} - ZZ^* > 0 \right\},$$

$$K_2 = \left\{ Z \in \mathbb{C}\left[m \times m\right] : I^{(m)} - Z\overline{Z} > 0, \forall Z' = Z \right\},$$

$$K_3 = \left\{ Z \in \mathbb{C}\left[m \times m\right] : I^{(m)} + Z\overline{Z} > 0, \forall Z' = -Z \right\},$$

$$K_4 = \left\{ z \in \mathbb{C}^n : \left|\langle z, z\rangle\right|^2 - 2|z|^2 + 1 > 0, \left|\langle z, z\rangle\right| < 1 \right\},$$

bu yerda $I^{(m)} -$ $m-$ tartibli birlik matritsa, $Z^* - Z'$ transponerlangan matritsaning kompleks qo'shmasidir ($H-$ Ermit matritsasi va musbat aniqlangan). Ushbu sohalarning barchasi markazi O nuqtada ($O-$nol matritsa) bo'lgan bir jinsli, simmetrik, qavariq, chegaralangan, to'la doiraviy sohalardir.

Yuqoridagi klassik sohalarning kompleks o'lchovlari mos ravishda mk, $\frac{m(m+1)}{2}$, $\frac{m(m-1)}{2}$ va n sonlarga teng bo'ladi.

Ma'lumki $B_{m,n}^{(1)}$, $B_{m,n}^{(2)}$ va $B_{m,n}^{(3)}$ – birinchi, ikkinchi va uchunchi tip matritsaviy sharlar mos ravishda quyidagi ko'rinishga ega:

$$B_{m,n}^{(1)}=\left\{(Z_1,\dots,Z_n)=Z\in\mathbb{C}^n\left[m\times m\right]:I-\left\langle Z,Z\right\rangle>0\right\},$$

$$B_{m,n}^{(2)}=\left\{Z\in\mathbb{C}^n\left[m\times m\right]:I-\left\langle Z,Z\right\rangle>0,\forall Z_v^{'}=Z_v,v=1,\dots,n\right\},$$

$$B_{m,n}^{(3)}=\left\{Z\in\mathbb{C}^n\left[m\times m\right]:I+\left\langle Z,Z\right\rangle>0,\forall Z_v^{'}=-Z_v,v=1,\dots,n\right\}.$$

$B_{m,n}^{(k)}$ matritsaviy sharlarning ostovlari (Shilov chegaralari) $X_{m,n}^{(k)}$ orqali belgilanadi, k=1,2,3,

$$X_{m,n}^{(1)}=\left\{Z\in\mathbb{C}^n\left[m\times m\right]:\left\langle Z,Z\right\rangle=I\right\},$$

$$X_{m,n}^{(2)}=\left\{Z\in\mathbb{C}^n\left[m\times m\right]:I=\left\langle Z,Z\right\rangle,\forall Z_v^{'}=Z_v,v=1,\dots,n\right\},$$

$$X_{m,n}^{(3)}=\left\{Z\in\mathbb{C}^n\left[m\times m\right]:I+\left\langle Z,Z\right\rangle=0,\forall Z_v^{'}=-Z_v,v=1,\dots,n\right\}.$$

Eslatma uchun $B_{1,1}^{(1)}$, $B_{1,1}^{(2)}$ va $B_{1,1}^{(3)}$ $\mathbb{C}$ komleks tekislikdagi birlik doiralardir $X_{1,1}^{(1)}$, $X_{1,1}^{(2)}$ va $X_{1,1}^{(3)}$ lar esa $\mathbb{C}$ komleks tekislikdagi birlik ayalanalardir.

$B_{m,n}^{(2)}$ **shar avtomorfizmi.**

$B_{m,1}^{(2)}$ avtomorfizm $P\in B_{m,1}^{(2)}$ nuqtani 0 nuqtaga akslantiradi va quyidagi ko'rinishga ega:

$$W=R(Z-P)(I-\bar{P}Z)^{-1}\bar{R}^{-1},$$

bu yerda R (1) shartni qanoatlantiruvchi $(m\times m)$ matritsa.

$$\bar{R}(I-\bar{P}P')R'=I \qquad (1)$$

Bizning maqsad ikkinchi tip matritsaviy sharning avtomorfizmini topish. Faraz qilaylik kerakli avtomorfizm ushbu ko'rinishda bo'lsin:

$$W_k=\left(A_{00}+\sum_{j=1}^{n}Z_jA_{j0}\right)^{-1}\left(A_{0k}+\sum_{j=1}^{n}Z_jA_{jk}\right),\ k=1,\dots,n. \qquad (2)$$

bu yerda A_{ij} $m-$tartibli kvadrat matritsa.

$n+1$ tartibli blok kvadrat matritsalarni quyidagi shaklda kiritamiz

$$A=\begin{pmatrix} A_{00} & A_{01} & \cdots & A_{0n} \\ A_{10} & A_{11} & \cdots & A_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ A_{n0} & A_{n1} & \cdots & A_{nn} \end{pmatrix},\ H=\begin{pmatrix} I^{(m)} & 0 & \cdots & 0 \\ 0 & -I^{(m)} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -I^{(m)} \end{pmatrix}.$$

(2) – akslantirish uchun ushbu teorema o'rinlidir.

Teorema-1. *Agar* A_{ij} $i,j=1,2,\ldots,n$ *koeffisientlar*

$$AHA^*=H,\ A_{sk}A'_{j0}=A_{j0}A'_{jk},\ s=0,\ldots,n;\ \ j,k=0,\ldots,n$$

munosabatni qanoatlantirsa, (2) – akslantirish $B^{(2)}_{m,n}$ *matritsaviy sharning avtomorfizmi bo'ladi.*

Endi $P=(P_1,\ldots,P_n)\in B^{(2)}_{m,n}$ nuqta va P nuqtani 0 nuqtaga o'tqazuvchi biror

$$W_k=R^{-1}\left(I^{(m)}-\langle Z,P\rangle\right)^{-1}\sum_{s=1}^{n}(Z_s-P_s)G_{sk} \tag{3}$$

akslantirish olamiz, bu yerda R,G_{sk} ixtiyoriy matritsalar.

Teorema-2. *(3)- ko'rinishdagi akslantirishlar ikinchi tip matritsaviy sharning avtomorfizmi bo'lishi uchun,* R va G matritsalar quyidagi munoasabatlarni qanoatlantirishi zarur va yetarlidir.

$$R^*\left(I^{(m)}-\langle P,P\rangle\right)R=I^{(m)},\ G^*\left(I^{(mn)}-P*P\right)G=I^{(mn)},$$

bu yerda G blok matritsa.

Xulosa qilib aytganda ushbu ishda ikkinchi tip matritsaviy shar avtomorfizmi bo'la oladigan akslantirishlar ko'rsatildi.

References:

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