# KO`P O`ZGARUVCHILI FUNKSIYALARNING O'ZGARUVCHILARI YANA FUNKSIYA BO'LADIGAN IFODALAR VA HOSILA UCHUN LAPLAS OPERATORIGA DOIR MISOLLAR

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**Annotatsiya:** Biz bu maqolada asosan ko`p o`zgaruvchili funksiyalarning differensiallari,Laplas operatorining ikki o`zgaruvchili funksiyaga tatbiqiga doir misollar hamda murakkab ikki o`zgaruvchili funksiyalarni birdan ortiq hosilalarini ko`rib chiqamiz.

**Kalit so`zlar:** funksiya,differensial,oshkormas funksiya,Laplas operatori,gradiyent.

## FUNKSIYA O`ZGARUVCHISI YANA FUNKSIYA KO`RINISHIDA BO'LADIGAN FUNKSIYALAR DIFFERENSIALLARI

Bizga ko`p o`zgaruvchili funksiyalarning differensiallari,ularning xususiy hosilalari ma`lum.Uch o`zgaruvchili funksiyaning differensiali u=f(x,y,z) funksiya uchun differensial shunday ko'rinishga ega bo'ladi.

$$df = \frac{\delta f}{\delta x_1} dx_1 + \frac{\delta f}{\delta x_2} dx_2 + \frac{\delta f}{\delta x_3} dx_3$$

Bunda berilgan funsiyaning differensialini topishimiz uchun u funksiyaga o`z o`zgaruvchisi bo`yicha xususiy hosilaga ega bo`lishi va xususiy hosilaga ega bo`lishi uchun uzluksiz bo`lishi kerak.Endilikda biz berilgan funksiyadagi argumentlarning o`zi ham qandaydir o`zgaruvchi yordamida qatnashishiga doir misol ko`ramiz.

1-MISOL.u=f($\mu, \delta, \vartheta$) funksiya berilgan bu yerda $\mu = a_1 x + b_1 y + c_1 z$ , $\delta = a_2 x + b_2 y + c_2 z, \vartheta = a_3 x + b_3 y + c_3 z$ ga teng bo`lsin.U holda bu funksiyaning birinchi va n-tartibli differensialini toping.

Bu funksiya uchta o`zgaruvchiga ega bo`lgan funksiya,differensialini topish uchun uning xususiy hosilalarini topish yetarli,bunda har bir o`zgaruvchi bo`yicha bu funksiyaning xususiy hosilalarini topsak, faqat argumentlarni o`zi qoladi.

du=$f'_1(a_1 dx + b_1 dy + c_1 dz) + f'_2(a_2 dx + b_2 dy + c_2 dz) + f'_3(a_3 dx + b_3 dy + c_3 dz)$ ga teng bo`ladi.Endi bu funksiyaning n-tartibli differensialini topsak,

du=dx($a_1 \frac{\partial}{\partial \mu} + a_2 \frac{\partial}{\partial \delta} + a_3 \frac{\partial}{\partial \vartheta}) + dy\left(b_1 \frac{\partial}{\partial \mu} + b_2 \frac{\partial}{\partial \delta} + b_3 \frac{\partial}{\partial \vartheta}\right) + dz\left(c_1 \frac{\partial}{\partial \mu} + c_2 \frac{\partial}{\partial \delta} + c_3 \frac{\partial}{\partial \vartheta}\right) f(\mu, \delta, \vartheta)$ holatga keltirsak,endi uning n-tartibli differensiali:

$d^n$u=$[dx(a_1\frac{\partial}{\partial\mu}+a_2\frac{\partial}{\partial\delta}+a_3\frac{\partial}{\partial\vartheta})+dy\left(b_1\frac{\partial}{\partial\mu}+b_2\frac{\partial}{\partial\delta}+b_3\frac{\partial}{\partial\vartheta}\right)+dz\left(c_1\frac{\partial}{\partial\mu}+c_2\frac{\partial}{\partial\delta}+c_3\frac{\partial}{\partial\vartheta}\right)]^n f(\mu,\delta,\vartheta)$ ko`rinishiga ega bo`ladi.

**LAPLAS OPERATORI.**

2-MISOL.Bizga u=f(r), r=$\sqrt{x^2+y^2+z^2}$ $\Delta u=F(r)$berilgan bo`lsa,mana shu funksiya uchun Laplas operatori ifodasini tuzing.

Yechish:

$\Delta u=\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}$ –berilgan ifodaga Laplas operatori deb ataladi.Bu funksiya murakkab ko`rinishda berilgani uchun funksiyaning har bir argumenti bo`yicha 1-tartibli va ikkinchi tartibli hosilalarini topib chiqamiz:

1.$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$=$f'(r)\left[\frac{x}{\sqrt{x^2+y^2+z^2}}+\frac{y}{\sqrt{x^2+y^2+z^2}}+\frac{z}{\sqrt{x^2+y^2+z^2}}\right]$ga teng bo`ladi.

$\Delta u=\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}=f''(r)+\frac{y^2+z^2}{\sqrt{(x^2+y^2+z^2)^3}}+\frac{x^2+z^2}{\sqrt{(x^2+y^2+z^2)^3}}+\frac{x^2+y^2}{\sqrt{(x^2+y^2+z^2)^3}}=$ $[r=\sqrt{x^2+y^2+z^2}]=f''(r)+\frac{2f'(r)}{r}$ ga teng bo`ladi.Demak bizga berilgan funksiyaning lopital operatori: $\Delta u=f''(r)+\frac{2f'(r)}{r}$ ga teng ekan.

3-MISOL.Quyidagi ayniyatni isbotlang:$\frac{\partial u}{\partial x}\cdot\frac{\partial^2u}{\partial x\partial y}=\frac{\partial u}{\partial y}\cdot\frac{\partial^2u}{\partial x^2}$ bunda u=$\varphi[x+\tau(y)]$

Isbot.Isbotlash uchun berilgan ko'rinishdagi hosilalarni topib olamiz.

1.$\frac{\partial u}{\partial x}=\varphi'[x+\tau(y)]$ 2. $\frac{\partial u}{\partial y}=\varphi'[x+\tau(y)]\tau'(y)\ \ \tau'(y)$ 3. $\frac{\partial^2u}{\partial x\partial y}=\varphi''[x+\tau(y)]\tau'(y)$

4.$\frac{\partial^2u}{\partial x^2}=\varphi''[x+\tau(y)]$ ga teng bo`ladi.$\frac{\partial u}{\partial x}\cdot\frac{\partial^2u}{\partial x\partial y}=\varphi'[x+\tau(y)]\ \cdot(\varphi''[x+\tau(y)]\tau'(y))$,$\frac{\partial u}{\partial y}\cdot\frac{\partial^2u}{\partial x^2}$φ"x+τ(y)τ'(y)]), ∂u/∂y·∂^2u/∂x^2= $[\varphi'[x+\tau(y)]\tau'(y)]\cdot\varphi''[x+\tau(y)]$ ga teng chiqadi,demak ayniyat isbotlandi.

Shuningdek,biz yuqorida aytib o`tgan lopital operatorimizni ikki argumentli uch argumentli hamda ko`p argumentli funksiyalarga tatbiq qilsak bo`ladi.Berilayotgan differensial ko'rinishdagi misollarimizning barchasida birinchi navbatda ularning har bir o`zgaruvchisi bo`yicha hosilasini topib olishimiz zarur,funksiya hosilaga ega bo`lishi uchun esa u uzluksiz va biror bir oraliqda aniqlangan bo`lishi zarur hamda yetarlilik shartini bilishimiz talaba qilinadi.

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