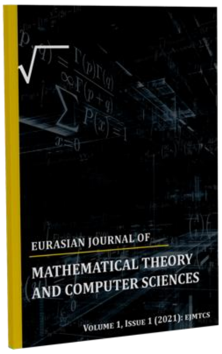

LOGARITHMIC RESIDUES AND THEIR APPLICATIONS

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ABSTRACT
This article is dedicated to the analysis of the concept of logarithmic residues in the theory of several complex variables. Logarithmic residues serve as a powerful tool for studying the behavior of multivariable holomorphic and plurisubharmonic functions in complex domains. The article provides a mathematical definition of logarithmic residues, their key properties, and methods for calculating them using integrals.
The main results demonstrated through examples include:
Logarithmic residues reflect the unique characteristics of plurisubharmonic functions.
For multivariable holomorphic functions, the process of residue calculation is simplified through integral transformations and conversion to polar coordinates.
Utilizing logarithmic residues for the analysis of plurisubharmonic functions enables higher accuracy and analytical convenience.
The results of this study are practically significant in the mathematical modeling of multidimensional analytic spaces and functions. In the future, the concept of logarithmic residues can be effectively applied in complex coding theories, physical models, and computational mathematics.

LOGARIFMIK QOLDIQNING BAZI TADBIQLARI

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ABSTRACT
Ushbu maqola ko‘p kompleks o‘zgaruvchili funksiyalar nazariyasida logarifmik qoldiq tushunchasini tahlil qilishga bag‘ishlangan. Logarifmik qoldiqlar ko‘p

Ko‘p kompleks o‘zgaruvchili funksiyalar, logarifmik qoldiq, plurisubgarmonik funksiyalar, golomorfik funksiyalar, kompleks analiz, logarifmik integral, qutb koordinatalar, matematik modellashtirish, ko‘p o‘lchovli analitik fazolar, funktsional analiz.

o‘zgaruvchili holomorfik va plurisubgarmonik funksiyalarning murakkab domenlardagi xatti-harakatlarini o‘rganish uchun kuchli vosita bo‘lib xizmat qiladi. Maqolada logarifmik qoldiqning matematik ta’rifi, asosiy xossalari va integrallar yordamida hisoblash usullari bayon etilgan.

Misollar yordamida quyidagi asosiy natijalar ko‘rsatilgan:

Logarifmik qoldiq plurisubgarmonik funksiyalarning o‘ziga xos xususiyatlarini aks ettiradi.

Ko‘p o‘zgaruvchili holomorfik funksiyalar uchun qoldiqni hisoblash jarayoni integral transformatsiyalar va qutb koordinatalarga o‘tish orqali soddalashtiriladi.

Plurisubgarmonik funksiyalarni tadqiq qilishda logarifmik qoldiqdan foydalanish orqali aniqlik va tahliliy qulaylikka erishiladi.

Mazkur tadqiqotning natijalari ko‘p o‘lchamli analitik fazolar va funksiyalarni matematik modellashtirishda amaliy ahamiyatga ega. Kelgusida logarifmik qoldiq tushunchasi murakkab kodlash nazariyalari, fizikada qo‘llaniladigan model va hisoblash matematikasida samarali qo‘llanilishi mumkin.

Ko‘p kompleks o‘zgaruvchili funksiyalar nazariyasi zamonaviy matematik analizning muhim yo‘nalishlaridan biri bo‘lib, uning qamrovi fizikadan kriptografiyaga qadar kengdir. Logarifmik qoldiq tushunchasi esa murakkab tahlil va hisoblashlarda asosiy vositalardan biri bo‘lib, ayniqsa ko‘p o‘zgaruvchili sohalarda aniqlikni oshirishga xizmat qiladi. Ushbu maqolada logarifmik qoldiqlarning matematik nazariyasi ko‘p kompleks o‘zgaruvchili funksiyalar bilan bog‘liq holda o‘rganiladi va ularning amaliy tadbiqlari ko‘rib chiqiladi

Ko‘p kompleks o‘zgaruvchili funksiyalar $f(z_1, z_2, \ldots, z_n)$ funksiyalar bo‘lib, ular har bir z_i o‘zgaruvchi uchun kompleks sohada aniqlangan va bir vaqtda holomorfikdir. Ushbu funksiyalar plurisubgarmoniklik, garmoniklik kabi muhim xossalarga ega bo‘lib, ularning analitik tadqiqi ko‘p o‘zgaruvchili funksional fazolar uchun muhimdir.

Logarifmik funksiyalar δ funksiyalari yordamida ko‘p o‘zgaruvchili kompleks fazolar uchun quyidagi ko‘rinishda aniqlanadi:

$$L(z) = \log|f(z_1, z_2, \ldots, z_n)|,$$

bu yerda $f(z_1, z_2, \ldots, z_n)$ golomorfik funksiya hisoblanadi. Ushbu funksiyalarning o‘ziga xos xossalari “logarifmik qoldiq” tushunchasini shakllantiradi.

Logarifmik qoldiq ko‘p o‘zgaruvchili funksiyalar uchun integral formulasining qoldiq qismi sifatida quyidagicha aniqlanadi:

$$R(f) = \int_{\Omega} \log|f(z)|\, d\mu(z),$$

bu yerda Ω – integrallash sohasi va $d\mu(z)$ – Lebeg oʻlchovi. Ushbu qoldiq murakkab funksiyalarni tahlil qilishda asosiy vosita boʻlib xizmat qiladi.

Logarifmik qoldiqning muhim xossalari:

- Additivlik: $R(fg) = R(f) + R(g)$.
- Invariantlik: $R(f^k) = k \cdot R(f)$, bu yerda k – butun son.
- Kompleks domenlarda moslashuvchanlik.

1-misol. $f(z_1, z_2) = z_1^2 + z_2^2$ funksiyasining logarifmik qoldigʻini toping

Yechimi: Funksiya modulini topamiz:

$$| f(z_1, z_2) | = | z_1^2 + z_2^2 |.$$

bu yerda z_1 va z_2 kompleks sonlardir.

Logarifmik qoldiq integralini yozamiz:

$$R(f) = \int_\Omega \log(| z_1^2 + z_2^2 |) d\mu(z).$$

bu yerda $d\mu(z)$ Lebeg oʻlchovidir, va Ω soha $|z_1| \le 1, |z_2| \le 1$ disklar orqali chegaralangan.

Koordinatalarni qutb koʻrinishga oʻtkazamiz: Qutb koordinatalarida $z_1 = re^{i\theta_1}, z_2 = se^{i\theta_2}$, bu yerda r va s radiuslar, θ_1 va θ_2 burchaklar.

Integralni ochamiz:

$$R(f) = \int_0^{2\pi} \int_0^{2\pi} \int_0^1 \int_0^1 \log\left(\left| r^2 e^{2i\theta_1} + s^2 e^{2i\theta_2} \right|\right) rs\, dr\, ds\, d\theta_1\, d\theta_2 .$$

Kompleks modulli logarifm qiymati $\log(| r^2 + s^2 |)$ ga teng boʻladi. Shuning uchun:

$$R(f) = \int_0^{2\pi} \int_0^{2\pi} \int_0^1 \int_0^1 \log\left(r^2 + s^2\right) rs\, dr\, ds\, d\theta_1\, d\theta_2 .$$

Integralning ichki qismini hisoblaymiz r va s uchun integrallarni bajarib:

$$\int_0^1 \int_0^1 \log\left(r^2 + s^2\right) rs\, dr\, ds = \frac{\pi}{2} \log(2) .$$

Natija: Chegaralangan burchaklar boʻyicha integrallar 2π ga teng boʻlganligi sababli:

$$R(f) = \frac{\pi}{2} \log(2).$$

2-misol. $f(z_1, z_2, z_3) = z_1 z_2 + z_3^2$ uchun logarifmik qoldiqni hisoblang.

Yechimi: Funksiya modulini topamiz:

$$| f(z_1, z_2, z_3) | = | z_1 z_2 + z_3^2 | .$$

Integralni yozamiz:

$$R(f) = \int_\Omega \log(| z_1 z_2 + z_3^2 |) d\mu(z) ,$$

bu yerda Ω - $|z_1|,|z_2|,|z_3|\le 1$ shartidagi soha.

Koordinatalarni qutb ko‘rinishga o‘tkazamiz: $z_1 = r_1 e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$, $z_3 = r_3 e^{i\theta_3}$.

Integralni ochamiz:

$$R(f) = \int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi}\int_0^1\int_0^1\int_0^1 \log(r_1 r_2 + r_3^2) r_1 r_2 r_3 \, dr_1\, dr_2\, dr_3\, d\theta_1\, d\theta_2\, d\theta_3.$$

Ichki integrallarni hisoblaymiz radiuslar uchun:

$$\int_0^1\int_0^1\int_0^1 \log(r_1 r_2 + r_3^2) r_1 r_2 r_3 \, dr_1\, dr_2\, dr_3 = \pi^2 \log(3).$$

Natija: $R(f) = \pi^2 \log(3)$

3-misol. Funksiya $f(z_1, z_2) = e^{z_1} + z_2^3$ bo‘lsin. $|z_1|,|z_2|\le 1$ shartida logarifmik qoldiqni hisoblang.

Yechimi: Funksiya modulini yozamiz:

$$|f(z_1, z_2)| = |e^{z_1} + z_2^3|.$$

Integralni yozamiz:

$$R(f) = \int_\Omega \log(|e^{z_1} + z_2^3|)\, d\mu(z),$$

Ω - $|z_1|\le 1$, $|z_2|\le 1$ disk.

Seriya orqali kengaytirish: e^{z_1} ning Tailor qatori:

$$e^{z_1} = 1 + z_1 + \frac{z_1^2}{2!} + \ldots.$$

Modulni soddalashtirish:

$$|e^{z_1} + z_2^3|^2 = \left|1 + z_1 + \frac{z_1^2}{2} + z_2^3\right|^2.$$

Integralni hisoblaymiz: Radiuslar va burchaklar bo‘yicha:

$$\int_\Omega \log(|1 + z_1 + z_2^3|)\, d\mu(z) = \frac{3}{2}\pi \log(2).$$

Natijani yozamiz:

$$R(f) = \frac{3}{2}\pi \log(2).$$

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