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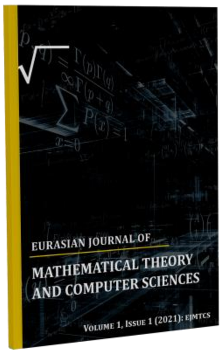

HIGH ACCURACY DIFFERENCE SCHEME FOR THE EQUATION OF DYNAMICS OF COMPRESSIVE STRATIFIED ROTATING FLUID

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ABSTRACT

Difference schemes of the finite difference method of high-order accuracy for the sixth-order Sobolev-type equation are constructed and investigated. In particular, the first boundary value problem for the wave equation of a compressible stratified rotating fluid is considered. First, approximation is performed only in spatial variables by the finite difference method, and the resulting system of high-dimensional ordinary differential equations is also approximated by this method. Using the method of energy inequalities, a priori estimates were obtained and, on their basis, theorems on the stability and convergence of the constructed difference schemes were proven; accuracy estimates were obtained for sufficient smoothness of the solution to the original initial boundary value problem. An algorithm for implementing difference schemes is proposed.

1. Introduction. In mathematical modeling of applied problems in complex many-sided fields, such as geophysics, oceanology, semiconductor physics, atmospheric physics, physics of magnetically ordered structures, associated with the wave propagation in media with a strong dispersion, non-classical high-order partial differential equations, called Sobolev-type equations, arise [1]–[3]. It is not always possible to find exact solutions to these equations, so they are mainly solved by numerical methods.

In [3]-[6], based on analytical methods, the problems of global and local solvability of initial boundary value problems for linear and nonlinear equations unsolved for the highest time derivative were considered. The solvability of such problems is also considered in [7]–[11], where theoretical results were obtained based on phase space methods developed by G.A. Sviridyuk.

Recently, numerous studies have been published on numerical solutions of initial boundary value problems for linear and nonlinear equations of Sobolev-type. In particular, in [2], [3], such equations were reduced to two equations using a certain function (one contains differentiation in time, the other - only in space) and then these equations were solved by the finite difference method on quasi-uniform grids. In [12], a mathematical model of ion-acoustic waves in a plasma in an external magnetic field and issues of unique solvability of the Cauchy-

Dirichlet problem were considered. A similar study of an optimal control problem for a given mathematical model was considered in [13], where an algorithm for a numerical solution was developed based on the modified Galerkin method and the Ritz method. In [14], a software package was proposed for the numerical solution to the Boussinesq-Love equation. In [15]-[17], similar problems were solved by the finite element method.

This study is devoted to the construction of difference schemes of high accuracy for the first initial boundary value problem for the equation of waves of a compressible stratified rotating fluid. The construction of difference schemes is performed based on the finite difference method for both variables. First, only spatial variables are approximated, resulting in a system of high-dimensional ordinary differential equations. Below, difference schemes of various orders of accuracy are considered for this system. Using the method of energy inequalities, A.A. Samarskii obtained various a priori estimates and, based on these estimates, the convergence and accuracy of difference schemes were proven with sufficient smoothness of the solution to the original initial boundary value problem. Algorithms for implementing difference schemes are proposed.

2. Statement of the problem. In domain

$$\bar{Q}_T = \left\{(x,t): x=(x_1,x_2,x_3)\in\bar{\Omega}=[0\le x_\alpha \le l_\alpha,\ \alpha=1,2,3],\ t\in[0,T]\right\}$$

we consider the initial boundary value problem for the equation of dynamics of compressible stratified rotating fluid in the following form [18]:

$$\frac{1}{c^2}\frac{\partial^4 u}{\partial t^4} = \frac{\partial^2}{\partial t^2}K_0 u + K_1 u + f(x,t),\quad (x,t)\in Q_T, \tag{1}$$

$$\left.\frac{\partial^k}{\partial t^k}u(x,t)\right|_{t=0} = u_{0,k},\quad k=\overline{0,3},\quad x\in\Omega, \tag{2}$$

$$u(x,t)|_{\Gamma} = \mu(t),\quad t\in[0,T], \tag{3}$$

where $K_0 = \Delta_3 - (\beta^2 + \alpha^2/c^2)$, $K_1 = \omega_0^2\Delta_2 + \alpha^2\partial^2/\partial x_3^2 - \alpha^2\beta^2$, $\Delta_3 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$, $\Delta_2 = \Delta_3 - \partial^2/\partial x_3^2$, ω_0^2 is the square of the Väisälä-Brent frequency, $u=(x,t)$ is the velocity of motion, c is the speed of sound, α,β are some constants, $Q_T = \Omega\times(0,T]$, $\Omega = \left\{0 < x_\alpha < l_\alpha,\ \alpha=1,2,3\right\}$ [1]. Here, $c\neq 0$, and for $c=\infty$ equation (1) is the equation of gravitational-gyroscopic waves, which is a mathematical model of linear internal waves in rotating ocean [4].

3. Discretization in space. Below, equation (1) is considered in the following form:

$$\frac{\partial^4 u}{\partial t^4} = \frac{\partial^2}{\partial t^2}K_0 u + K_1 u + f(x,t), \tag{4}$$

where $K_0 = c^2\Delta_3 - (c^2\beta^2 + \alpha^2)$, $K_1 = c^2\omega_0^2\Delta_2 + c^2(\alpha^2\partial^2/\partial x_3^2 - \alpha^2\beta^2)$.

Let us construct subspace $H_h \subset H$ that approximates Hilbert space H with the corresponding scalar product and norm. We introduce into $\overline{\Omega}$ grid (uniform in each direction) $\overline{\omega}_h = \overline{\omega}_{h_1} \times \overline{\omega}_{h_2} \times \overline{\omega}_{h_3}$, where $\overline{\omega}_{h_\alpha} = \{x_\alpha = i_\alpha h_\alpha, i_\alpha = \overline{0, N_\alpha}, \ h_\alpha = l_\alpha / N_\alpha\}, \alpha = 1,2,3.$

Here $\overline{\omega}_h = \omega_h + \gamma_h$. We define subspace $H_h = \overset{\circ}{W}{}_2^1(\omega_h)$ - the space of grid functions $v(x_1, x_2, x_3)$ with norm $\|v\|_1^2 = \sqrt{\sum_{i_1}^{N_1} \sum_{i_2}^{N_2} \sum_{i_3}^{N_3} h_1 h_2 h_3 \left[(v_{\bar{x}_1})^2 + (v_{\bar{x}_2})^2 + (v_{\bar{x}_3})^2 \right]} \le M$, where constant M does not depend on h_1, h_2, h_3. Here $v = v(i_1 h_1, i_2 h_2, i_3 h_3)$,

$$v_{\bar{x}_1} = \left[v(i_1 h_1, i_2 h_2, i_3 h_3) - v((i_1 - 1) h_1, i_2 h_2, i_3 h_3) \right] / h_1,$$

$$v_{\bar{x}_2} = \left[v(i_1 h_1, i_2 h_2, i_3 h_3) - v(i_1 h_1, (i_2 - 1) h_2, i_3 h_3) \right] / h_2,$$

$$v_{\bar{x}_3} = \left[v(i_1 h_1, i_2 h_2, i_3 h_3) - v(i_1 h_1, i_2 h_2, (i_3 - 1) h_3) \right] / h_3.$$

Approximating operators K_0 and K_1 by difference relations on the indicated grids, we obtain the Cauchy problem for a system of ordinary fourth-order differential equations:

$$D \frac{d^4 u_h}{dt^4} + B \frac{d^2 u_h}{dt^2} + A u_h(t) = f_h, \quad \frac{d^k u_h}{dt^k}(0) = u_{0,k,h}, \ k = \overline{0,3}, \qquad (5)$$

where D, B and A are linear constant operators from $H_h \to H_h$, $D^* = D > 0$, $B^* = B \ge 0$, $A^* = A > 0$, $\forall\, t \ge 0$, $u_h = u_h(t) \in H_h$, $f_h = f_h(t) \in H_h$. Here

$$B = c^2 \Lambda - (c^2 \beta^2 + \alpha^2) E, \quad A = c^2 \alpha^2 (\Lambda - \beta^2 E), \quad \Lambda = \sum_{\alpha=1}^{3} \Lambda_\alpha, \qquad (6)$$

$\Lambda_m u_h = -u_{h, x_m \bar{x}_m}$, $m = 1,2,3$, u_h is the function value at fixed node $x = (i_1 h_1, i_2 h_2, i_3 h_3)$,

$$u_{h, x_1 \bar{x}_1} = (u_h((i_1 + 1) h_1, i_2 h_2, i_3 h_3) - 2 u_h(i_1 h_1, i_2 h_2, i_3 h_3) + u_h((i_1 - 1) h_1, i_2 h_2, i_3 h_3)) / h_1^2,$$

$$u_{h, x_2 \bar{x}_2} = (u_h(i_1 h_1, (i_2 + 1) h_2, i_3 h_3) - 2 u_h(i_1 h_1, i_2 h_2, i_3 h_3) + u_h(i_1 h_1, (i_2 - 1) h_2, i_3 h_3)) / h_2^2,$$

$$u_{h, x_3 \bar{x}_3} = (u_h(i_1 h_1, i_2 h_2, (i_3 + 1) h_3) - 2 u_h((i_1 h_1, i_2 h_2, i_3 h_3)) + u_h(i_1 h_1, i_2 h_2, (i_3 - 1) h_3)) / h_3^2.$$

Operators B and A approximate operators K_0 and K_1 with the second order, respectively, i.e. $O(|h|^2), \ |h| = \sqrt{h_1^2 + h_2^2 + h_3^2}$, $D = E$.

4. Discretization in time. We introduce uniform grid $\bar{\omega}_\tau = \{t_n = n\tau,\ n = 0,1,...;\ \tau > 0\}$ on segment $[0,T]$. Let y approximate u_h. In [19], for problem (5) with operators (6), a difference scheme of the fourth-order approximation was constructed on this grid:

$$\bar{D}y_{\bar{t}t\bar{t}t} + \bar{B}y_{\bar{t}t} + Ay = \bar{\varphi},\ \ t_n \in \omega_\tau, \quad (7)$$

$$y^0 = u_{0,0},\ y^1 = \bar{u}_{0,1},\ y^2 = \bar{u}_{0,2},\ y^3 = \bar{u}_{0,3}, \quad (8)$$

where

$$\bar{D} = D + (\tau^2/12)B,\ \ \bar{B} = B + (\tau^2/6)A, \quad (9)$$

$$y_{\bar{t}t\bar{t}t} = (y^{n+2} - 4y^{n+1} + 6y^n - 4y^{n-1} + y^{n-2})/\tau^4, \qquad y_{\bar{t}t} = (y^{n+1} - 2y^n + y^{n-1})/\tau^2,$$

$$y^n = y(t_n),\ \ y^{n\pm1} = y(t_n \pm \tau),\ \ y^{n\pm2} = y(t_n \pm 2\tau),\ \ \bar{\varphi} = \varphi + (\tau^2/6)\partial^2 f/\partial t^2,$$

$$\bar{u}_{0,1} = u_{0,1} + 0.5\tau[E - (\tau^2/12)D^{-1}B]u_{0,2} + (\tau^2/6)u_{0,3} + (\tau^3/24)D^{-1}[f(0) - Au_{0,0}],$$

$$\bar{u}_{0,2} = u_{0,2} + \tau u_{0,3} + (\tau^2/2)D^{-1}[f(0) - Bu_{0,2} - Au_{0,0}] +$$
$$+(\tau^3/4)D^{-1}[\dot{f}(0) - B\dot{u}_{0,2} - A\dot{u}_{0,0}],$$

$$\bar{u}_{0,3} = u_{0,3} + (3\tau/2)D^{-1}[f(0) - Bu_{0,2} - Au_{0,0}] + (5\tau^2/4)D^{-1}[\dot{f}(0) - B\dot{u}_{0,2} - A\dot{u}_{0,0}] +$$

5. Convergence of the scheme. The following assertion holds.

Theorem 1 [19]. Let $D^* = D > 0$, $B^* = B \geq 0$, $A^* = A > 0$ and the following stability condition be satisfied

$$\bar{D} \geq (\tau^4/4)A. \quad (10)$$

Then the solution to the difference scheme (7), (8) with operators (9) converges to a smooth solution of the original problem (5) and the following accuracy estimate holds:

$$\|y(t_n) - u(t_n)\| \leq O(\tau^4),\ t_n \in \bar{\omega}_\tau.$$

Therefore, based on this theorem, we obtain the following result.

Theorem 2. Let $D^* = D > 0$, $B^* = B \geq 0$, $A^* = A > 0$ and stability condition (10) be satisfied. Then, the solution to the difference scheme (7), (8) with operators (9) converges to a smooth solution of the original problem (1)-(3) or (4), (2), (3) and the following accuracy estimate holds for its solution:

$$\|y(x_i,t_n) - u(x_i,t_n)\| \leq O(|h|^2 + \tau^4),\ \ x_i \in \bar{\omega}_h,\ \ t_n \in \bar{\omega}_\tau.$$

6. Schemes with weights. Based on the difference scheme (7), (8) with operators (9), we consider a family of difference schemes with weights

$$\bar{D}y_{\bar{t}t\bar{t}t} + \bar{B}y_{\bar{t}t}^{(\sigma_1,\sigma_2)} + Ay^{(\sigma_3,\sigma_4)} = \varphi,\ \ t_n \in \omega_\tau. \quad (11)$$

Here $y^{(\sigma_1,\sigma_2)} = \sigma_1 \hat{y} + (1 - \sigma_1 - \sigma_2)y + \sigma_2 \check{y}$, $y^{(\sigma_3,\sigma_4)} = \sigma_3 \hat{y} + (1 - \sigma_3 - \sigma_4)y + \sigma_4 \check{y}$,

where $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ are some constants of the scheme with weights, the presence of which allows us to select various explicit and implicit schemes and adjust their accuracy in space.

Let us study the stability and convergence of scheme (11) with initial conditions (8). To do this, we reduce (11) to canonical form [20]. The following transformation on scheme (11) is performed:

$$(\bar{D}+\tau^2\sigma_1\bar{B})y^{n+2}-[4\bar{D}-\tau^2(1-\sigma_1-\sigma_2)\bar{B}+2\tau^2\sigma_1\bar{B}-\tau^4\sigma_3 A]y^{n+1}+$$
$$+[6\bar{D}+\tau^2\sigma_2\bar{B}-2\tau^2(1-\sigma_1-\sigma_2)\bar{B}+\tau^2\sigma_1\bar{B}+\tau^4(1-\sigma_3-\sigma_4)A]y^n-$$
$$-[4\bar{D}+2\tau^2\sigma_2\bar{B}-\tau^2(1-\sigma_1-\sigma_2)\bar{B}-\tau^4\sigma_4 A]y^{n-1}+(\bar{D}+\tau^2\sigma_2\bar{B})y^{n-2}=\tau^4\bar{\varphi}. \quad (12)$$

Let $y=y^{n+2}$ in (12). Then (12) has the following form:

$$\mathrm{B}_4 y^{n+4}+\mathrm{B}_3 y^{n+3}+\mathrm{B}_2 y^{n+2}+\mathrm{B}_1 y^{n+1}+\mathrm{B}_0 y^n=\tau^4\bar{\varphi}^n,$$

where

$$\mathrm{B}_4=\bar{D}+\tau^2\sigma_1\bar{B},\ \mathrm{B}_3=-[4\bar{D}-\tau^2(1-\sigma_1-\sigma_2)\bar{B}+2\tau^2\sigma_1\bar{B}-\tau^4\sigma_3 A],$$
$$\mathrm{B}_2=6\bar{D}+\tau^2\sigma_2\bar{B}-2\tau^2(1-\sigma_1-\sigma_2)\bar{B}+\tau^2\sigma_1\bar{B}+\tau^4(1-\sigma_3-\sigma_4)A, \quad (13)$$
$$\mathrm{B}_1=-[4\bar{D}+2\tau^2\sigma_2\bar{B}-\tau^2(1-\sigma_1-\sigma_2)\bar{B}-\tau^4\sigma_4 A],\ \mathrm{B}_0=\bar{D}+\tau^2\sigma_2\bar{B}.$$

Now, similarly to [20], we write scheme (12) in the following canonical form:

$$My_{\bar{t}}+\tau^2 Ry_{\overset{\circ}{t}\overset{\circ}{t}}+\tau^3 Py_{\bar{t}\bar{t}\overset{\circ}{t}}+\tau^4 Qy_{\bar{t}\bar{t}t\bar{t}}+\mathrm{A}y=\tau^4\bar{\varphi}, \quad (14)$$

where

$$M=\tau(2\mathrm{B}_4+\mathrm{B}_3-\mathrm{B}_1-2\mathrm{B}_0),\ R=2\mathrm{B}_0+0.5(\mathrm{B}_1+\mathrm{B}_3)+2\mathrm{B}_4,$$
$$P=0.5(\mathrm{B}_1-\mathrm{B}_3),\ Q=-(1/8)(\mathrm{B}_1+\mathrm{B}_3),\ \mathrm{A}=\mathrm{B}_4+\mathrm{B}_3+\mathrm{B}_2+\mathrm{B}_1+\mathrm{B}_0.$$

Hence, considering (13), we obtain

$$M=\tau^5(\sigma_3-\sigma_4)A,\ R=\tau^2[1-2(\sigma_1+\sigma_2)]\bar{B}+0.5\tau^4(\sigma_4+\sigma_3)A,$$
$$P=-(\sigma_2-\sigma_1)\tau^2\bar{B}+0.5\tau^4(\sigma_4-\sigma_3)A,$$
$$Q=\bar{D}-(\tau^2/4)[1-2(\sigma_2+\sigma_1)]\bar{B}-(\tau^4/8)(\sigma_4+\sigma_3)A,\ \mathrm{A}=\tau^4 A.$$

Let $\sigma_1=\sigma_2=\sigma$, $\sigma_3=\sigma_4=\theta$, then $M=P=0$. Consequently, after elementary calculations, from (14) we obtain

$$\tilde{Q}y_{\bar{t}\bar{t}t\bar{t}}+\tilde{R}y_{\overset{\circ}{t}\overset{\circ}{t}}+Ay=\bar{\varphi}, \quad (15)$$

where

$$\tilde{Q}=\bar{D}-(\tau^2/4)(1-4\sigma)\bar{B}-(\tau^4/8)\theta A,\ \tilde{R}=\tau^2(1-4\sigma)\bar{B}+\tau^4\theta A.$$

According to Theorem 2 from [20, p. 276], there is an a priori estimate based on the initial data ($\varphi=0$)

$$\|Y^{n+1}\|_{\bar{\mathrm{A}}}\le\|Y^n\|_{\mathrm{A}}, \quad (16)$$

if the following conditions are met:

$$\mathrm{Re}M\ge 0,\ \mathrm{A}\ge 0,\ R-4Q-\mathrm{A}\ge 0,\ \mathrm{A}+16Q\ge 0. \quad (17)$$

Here

$$\|Y^n\|^2_{\bar{A}} = (1/16)\|y^n + y^{n+1} + y^{n+2} + y^{n+3}\|^2_{A} + (1/16)\Big[\|y^{n+3} + y^{n+2} - y^{n+1} - y^n\|^2_{R-4Q-A} +$$

$$+\|y^{n+3} - y^{n+2} - y^{n+1} + y^n\|^2_{R-4Q-A}\Big] + (1/16)\|y^{n+3} - y^{n+2} + y^{n+1} - y^n\|^2_{A+16Q}. \quad (18)$$

Let us check the fulfillment of conditions (17). The first condition $\mathrm{Re}\,M \geq 0$ is satisfied since $M = 0$. The second condition $A \geq 0$ is satisfied since $A > 0$. Condition $R - 4Q - A \geq 0$ will be satisfied if

$$4\bar{D} + \tau^4(1-2\sigma)A \leq 2\tau^2(1-4\sigma)\bar{B}, \quad (19)$$

and, finally, condition $A + 16Q \geq 0$ will be satisfied if

$$16\bar{D} + \tau^4(1-2\theta)A \geq 4\tau^2(1-4\sigma)\bar{B}. \quad (20)$$

Conditions (19) and (20) will be satisfied if $\theta \leq 1/2$, $\sigma \leq 1/4$, $\bar{D} \geq (\tau^4/4)A$ or

$$\theta \leq 1/2, \quad \sigma \leq 1/4, \quad D \geq (\tau^4/4)A, \quad (21)$$

which are conditions for the stability of scheme (11), (8).

Thus, the following theorem is proven.

Theorem 3. Let $D^* = D > 0$, $B^* = B \geq 0$, $A^* = A > 0$ and condition (21) be satisfied. Then, to solve the difference scheme (11), (8), an a priori estimate based on the initial data (16) is true.

To prove the stability on the right-hand side of scheme (11), (8), we present it in the form of an equivalent two-layer scheme in space H^4 [20]:

$$\mathbb{C}y_t + \mathbb{Q}y = \phi,$$

where $y_t = \left\{ y_{\bar{t}},\ \tau y_{\overset{\circ\circ}{tt}},\ (\tau^2/2)y_{\bar{t}\overset{\circ\circ}{tt}},\ (\tau/2)y_{\overset{\circ\circ}{tt}} + (\tau^3/8)y_{\bar{t}t\bar{t}t} \right\}$, $\phi = \{\varphi, 0, 0, 0\}$,

$$\mathbb{Q} = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & R-2Q-A & 0 & 0 \\ 0 & 0 & R-2Q-A & 0 \\ 0 & 0 & 0 & A+16Q \end{pmatrix},$$

Based on Theorem 4 from [20, p. 284], the following assertion holds.

Theorem 4. Let $D^* = D > 0$, $B^* = B \geq 0$, $A^* = A > 0$ and the following operator inequalities be satisfied:

$$\mathrm{Re}\,M \geq 0, \quad A > 0, \quad R - 4Q - A > 0, \quad A + 16Q > 0. \quad (22)$$

Then, to solve the difference scheme (15), (8), the following a priori estimate is valid:

$$\left\|Y^{n+1}\right\|_{\bar{A}} \le \left\|Y^{n}\right\|_{\bar{A}} + \left\|\bar{\varphi}^{0}\right\|_{\bar{A}^{-1}} + \left\|\bar{\varphi}^{n}\right\|_{\bar{A}^{-1}} + \sum_{k=1}^{n} \tau \left\|\bar{\varphi}_{\bar{t}}^{k}\right\|_{\bar{A}^{-1}},$$

where $\left\|Y^{n}\right\|_{\bar{A}}^{2}$ is calculated according to (18).

Let us check the fulfillment of conditions (22). The first two conditions are satisfied since $M = 0$ and $A = A^* > 0$, other conditions will be satisfied if inequalities (21) hold.

Based on Theorems 3 and 4, the following result holds.

Theorem 5. Let $D^* = D > 0$, $B^* = B \ge 0$, $A^* = A > 0$ and condition (21) be satisfied. Then the solution to the difference scheme (15), (8) converges to a smooth solution of the original problem (1)-(3) and the following accuracy estimate holds

$$\left\|y(x_i,t_n) - u(x_i,t_n)\right\| \le O(h^2 + \tau^4), \quad x_i \in \bar{\omega}_h, \ t_n \in \bar{\omega}_\tau.$$

7. Higher accuracy on spatial variables. If the solution to the original differential problem has the necessary smoothness in spatial variables, then difference operators of high-order approximation can be constructed. Obtaining difference operators with a higher order of approximation can be achieved in various ways. For example, operators of the difference scheme (15), (8) $\tilde{Q}$, $\tilde{R}$, and A are chosen in the following form ($\varphi \equiv 0$):

$$\tilde{Q} = \bar{D} - \frac{\tau^2}{4}\left(\bar{B} - \sigma \sum_{m=1}^{3} \frac{h_m^2}{12k_m} A_m\right), \quad \tilde{R} = \bar{B} + \tau^2 \sigma \sum_{m=1}^{3} \frac{h_m^2}{12k_m} A_m,$$

$$A = \sum_{m=1}^{3} A_m - \sum_{\substack{m,n=1 \\ m \ne n}}^{3} \frac{h_m^2}{12k_m} A_m A_n, \qquad (23)$$

where $A_m y = -\Lambda_m y$. Consequently, difference operators $\tilde{Q}$, $\tilde{R}$, and A in (23) approximate differential operators with the fourth order of approximation error, i.e. $O(|h|^4)$, $|h| = \sqrt{h_1^2 + h_2^2 + h_3^2}$.

8. Algorithms for implementing the scheme. Scheme (11) for $\sigma_1 = \sigma_2 = \sigma$, $\sigma_3 = \sigma_4 = \theta$ has the following form:

$$\bar{D} y_{\bar{t}t\bar{t}t} + \bar{B} y_{\bar{t}t}^{(\sigma)} + A y^{(\theta)} = \varphi, \quad t_n \in \omega_\tau,$$

where $y^{(\sigma)} = \sigma \hat{y} + (1 - 2\sigma) y + \sigma \check{y}$, $y^{(\theta)} = \theta \hat{y} + (1 - 2\theta) y + \theta \check{y}$. If $\sigma = \theta = 0$, then we obtain the explicit scheme (7), (8), implemented directly, and for other values of σ, we obtain an implicit scheme, implemented by the sweep method. Numerical calculations can be performed using the high-accuracy scheme (15), (8) with operators (23).

9. Conclusions. A boundary value problem for the equation of dynamics of a compressible stratified rotating fluid was considered. Based on the finite difference method, parametric difference schemes of high-order accuracy in time were constructed and studied. The presence of parameters in the scheme allows for regularization of schemes to optimize the implementation algorithm and the accuracy of the scheme. The corresponding a priori estimates were obtained and, on their basis, theorems on the rate of convergence and accuracy of the constructed algorithms were proven with sufficient smoothness of the solutions to the original differential problem. Based on these advantages, it is possible to study other boundary value problems, in particular, nonlocal boundary value problems. Moreover, these results can be extended to loaded equations with local and nonlocal boundary conditions.

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