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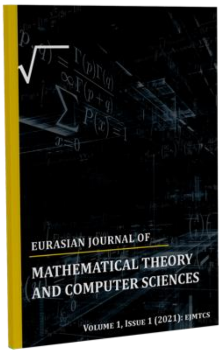

FINDING THE OPTIMAL SOLUTION OF A TOURISM COMPANY'S TOTAL COSTS FOR TRAVEL BETWEEN COUNTRIES WITH THE HELP OF THE TRAVELER'S PROBLEM (EXAMPLE OF "FRESH TOUR" COMPANY)

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https://doi.org/10.5281/zenodo.14259035

ARTICLE INFO

Received: 25th November 2024
Accepted: 29th November 2024
Online: 30th November 2024

KEYWORDS

Traveling salesman problem, method of networks and limits, optimal route, incomplete routes, incomplete cycle, the given table.

ABSTRACT

In this article, the formula of the travel agent problem of total expenses for trips between countries of the "Fresh tour" tourism firm, methods of solving it, and its practical application are considered. This article provides a detailed analysis of the theoretical foundations and practical solutions to this problem, as well as points out perspectives for future research.

Introduction

The traveling salesman problem is a classic problem that holds an important place in the fields of graph theory and optimization. In this problem, a merchant (traveler) must visit a set of cities, entering each city once and returning to the starting city at the end. The goal is to keep distances between cities to a minimum. Find the cycle of visiting n given cities in the shortest time (distance, cost) by visiting each of them only once. In this case, the number of cycles is at most $(n-1)!$ will be This problem is related to the problem of finding a Hamilton cycle of minimum length. The "Networks and Boundaries" method can be used to solve the traveling salesman problem. This method is carried out with the help of a connected graph without cycles and edges, and with the help of tables.

Literature analysis

The traveling salesman problem is studied as a classical problem of mathematical optimization, and various algorithms for its solution are proposed. "The Traveling Salesman Problem: A Computational Study" by D.L. Applegate, R.E. Bixby, V. Chvatal, W.J. Cook. This work is an important source for studying the theoretical and practical methods of solving the traveler's problem. "Transportnye zadachi i puti ix reshenia" — N.P. Buslenko, V.V. Krinitsyn. Algorithmic approaches to solving transport problems and methods of optimizing the commuter problem are considered in this work. M. Tokhtasinov. In the process research book, solutions to the traveling salesman problem and some issues are considered.

Research methodology

The main goal of the study is to determine the optimal route for tourism companies to reduce the costs of international travel. The research was carried out in cooperation with the tourist company "Fresh Tour". First, information such as distance between countries, types of transport and their costs are collected. Then, a cost matrix is created for each route. In order to minimize these costs, the classical commuter problem model is chosen. Traditional methods such as Genetic Algorithm or Travelling salesman problem are used to solve the problem. The resulting solutions are analyzed by evaluating total costs and efficiency for tourism firms. At the end of the study, conclusions and recommendations are given to reduce costs for tourism firms by recommending economically efficient routes.

Analyzes and results

We introduce the concept of bringing a table. To do this, the rows of the table are first brought, that is, the smallest of the row is subtracted from the elements of each row of the table. After that, the same operation is performed in relation to the table columns, and the table columns are listed. A table in which all rows and columns are listed is called a listed table. The sum of the smallest elements of the rows and columns of the table is denoted by h, and it is called the multiplication coefficient of the table. Consider the following problem.

Problem. According to the information of the tourist company "Fresh Tour", finding the optimal solution for a one-day trip to the countries of Uzbekistan, Malaysia, Thailand, Singapore and Bali using the Travelling Salesman problem. (Data as of October 7, US dollars ($) entered on a monetary unit basis.)

	Uzbekistan	**Malaysia**	**Thailand**	**Singapore**	**Bali**
Uzbekistan	∞	368	344	404	569
Malaysia	471	∞	49	62	59
Thailand	467	43	∞	89	80
Singapore	567	57	125	∞	94
Bali	924	49	85	93	∞

Table 1.

Solution: To quote the rows of Table 1, we write the smallest element of the corresponding row to its right.

	Uzbekistan	**Malaysia**	**Thailand**	**Singapore**	**Bali**	The smallest elements in a row
Uzbekistan	∞	368	344	404	569	344
Malaysia	471	∞	49	62	59	49
Thailand	467	43	∞	89	80	43
Singapore	567	57	125	∞	94	57
Bali	924	49	85	93	∞	49

By subtracting the smallest elements in each row from the row elements, we get the following table 2

	Uzbekistan	Malaysia	Thailand	Singapore	Bali
Uzbekistan	∞	24	0	60	225
Malaysia	422	∞	0	13	10
Thailand	424	0	∞	46	37
Singapore	510	0	68	∞	37

Table 2.

In order to list the columns of the resulting table 2, the smallest element of the corresponding column is written under the table.

	Uzbekistan	Malaysia	Thailand	Singapore	Bali
Uzbekistan	∞	24	0	60	225
Malaysia	422	∞	0	13	10
Thailand	424	0	∞	46	37
Singapore	510	0	68	∞	37
Bali	875	0	36	44	∞
The smallest elements in a column	422	0	0	13	10

By subtracting the smallest elements in each column from the column elements, we get Table 3 below.

	Uzbekistan	Malaysia	Thailand	Singapore	Bali
Uzbekistan	∞	24	0	47	215
Malaysia	0	∞	0	0	0
Thailand	2	0	∞	33	27
Singapore	88	0	68	∞	27
Bali	453	0	36	31	∞

Table 3.

Table 3 has at least one zero element in each row and column. The coefficient h of the table in question is equal to the following number:

$$h = 344 + 49 + 43 + 57 + 49 + 422 + 0 + 0 + 13 + 10 = 987$$

In general, the method of networks and boundaries consists of two important steps: 1) network; 2) determination of lower limits.

During the solution of the problem, both stages are carried out in parallel. To carry out these steps, you need to do the following in sequence:

a) bring the initial schedule;

b) determination of the coefficient h;

c) determine the level of zero elements of the given table;
d) perform branching based on these levels;
e) determining the lower limits of the cycles that form branching results;
f) reduce table size to one;
g) avoiding the formation of incomplete cycles;
h) continue this process until a (2×2) table is formed;
i) determine the cycle corresponding to the last network result;
j) compare all thresholds (grades);
k) if necessary, continue branching by restoring the schedule corresponding to the minimum marginal result.

In this method, all calculations are performed using a given table, and the results are displayed on a separate graph. At the end of this process, the perfect (lowest cost) cycle is determined.

The graph consists of interconnected circles, each of which defines a set of cycles with a certain property. The boundary numbers written next to these circles indicate the lower limit of the costs corresponding to the cycles belonging to this circle. The initial part of the graph looks like Figure 1. Here, the first initial circle defines the set containing all cycles and indicates that the cost of an arbitrary cycle cannot be less than the number h. In the example above, $h = 987$, which means that there is no cycle with a cost less than 987.The degrees of the zero elements of the given table 3 are defined for branching the set of all cycles. For example, to find the level of $c_{UT} = 0$ in the Uzbekistan row, Thailand column with zero element in Table 3, the smallest element in the Uzbekistan row is 24, and the Thailand column is the smallest element in the Malaysia row. The number 0 is added and the resulting number 24 is written as the power of this zero. Similarly, to find the degree of $c_{MU} = 0$, the smallest element in the row of Malaysia is added to 0, the smallest element in the column of Uzbekistan is 2, and the resulting number 2 is written as the degree of $c_{MU} = 0$ is placed. Using this method, the levels of all zero elements of Table 3 are determined.

	Uzbekistan	**Malaysia**	**Thailand**	**Singapore**	**Bali**
Uzbekistan	∞	24	$0^{(24)}$	47	215
Malaysia	$0^{(2)}$	∞	$0^{(0)}$	$0^{(31)}$	$0^{(27)}$
Thailand	2	$0^{(2)}$	∞	33	27
Singapore	88	$0^{(27)}$	68	∞	27
Bali	453	$0^{(31)}$	36	31	∞

The row i and column j with the largest zero are found and branched according to (i, j). Therefore, if there are several high-order zeros, an arbitrary one of them is selected. Here, the circle on the right represents the set of all cycles that include the transition from city i to city j and is denoted by (i, j), while the circle on the left, on the contrary, is the cycle from city i to city j denotes the set of routes that do not contain a tooth, and it is denoted by (j, i).

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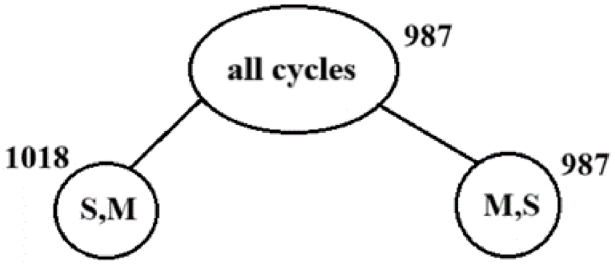

Figure 1.

The element with the highest rank of 31 is $c_{MS} = 0$ and $c_{BM} = 0$. We can choose any one of these elements. For example, $c_{BM} = 0$, so the branching graph will look like Figure 1. Next to the left circle, the number 1018 is written by adding the highest level of zero to $h = 987$ (h_2). The Bali row and Malaysia column of Table 3 are dropped (thus reducing the size of the table by one) to determine the lower bound of costs corresponding to the right-hand box. In this case, it should be noted that the serial numbers of the cities must be preserved (written), otherwise confusion will arise. After that, all incomplete cycles are forbidden to form, for example $i \to j \to i$ (the symbol $i \to j$ means the transition from $i -$ city to $j -$ city) is lost, for which c_{ji} the element is replaced by the ∞ symbol.

	Uzbekistan	**Thailand**	**Singapore**	**Bali**
Uzbekistan	∞	0	47	215
Malaysia	0	0	0	∞
Thailand	2	∞	33	27
Singapore	88	68	∞	27

Table 4.

After that, a new table is created, and its conversion coefficient is added to h, which is the previous conversion coefficient, and written (h_1).

	Uzbekistan	**Thailand**	**Singapore**	**Bali**	The smallest elements in a row
Uzbekistan	∞	0	47	215	0
Malaysia	0	0	0	∞	0
Thailand	2	∞	33	27	2
Singapore	88	68	∞	27	27

	Uzbekistan	**Thailand**	**Singapore**	**Bali**
Uzbekistan	∞	0	47	215
Malaysia	0	0	0	∞
Thailand	0	∞	31	25
Singapore	61	41	∞	0

	Uzbekistan	Thailand	Singapore	Bali
Uzbekistan	∞	$0^{(47)}$	47	215
Malaysia	$0^{(0)}$	$0^{(0)}$	$0^{(31)}$	∞
Thailand	$0^{(31)}$	∞	31	25
Singapore	61	41	∞	$0^{(66)}$

Table 5.

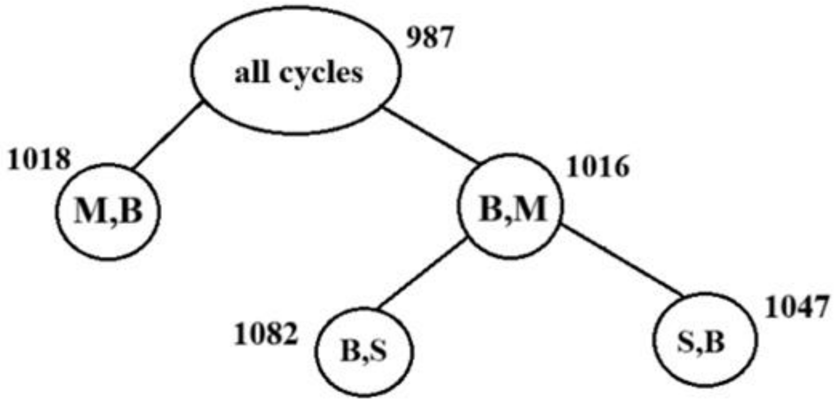

Figure 2.

As can be seen from the above tables, the multiplication factor is $27 + 2 = 29$. From this, $h_1 = 987 + 29 = 1016$. Table 5 is the given table.

The branching point is selected to the right (according to the rule of walking to the right) to determine the branching pair (k, l), the degrees of the zeros of the last table 5 are counted, and the pair (k, l) is separated in memory of the largest degree of them, and the branching is performed is increased (Fig. 2). In this case, the value of the border (h_4) corresponding to the left circle with the symbol $((l, k))$ is determined by adding the number with the greatest degree of zero to h_1.

To find the limit (h_3) corresponding to the right circle with the sign (k, l), k-row and l-column are removed from the last table (deleted) and incomplete cycles are generated using the sign ∞ is prohibited. After that, the multiplication factor of the resulting table is added to h_1 and written next to the right circle.

(k, l) taken as a pair (Singapore, Bali). Then we will have a suitable graph (Fig. 2). In this case, the lower limit of the left circle is equal to $h_4 = 1016 + 66 = 1082$.

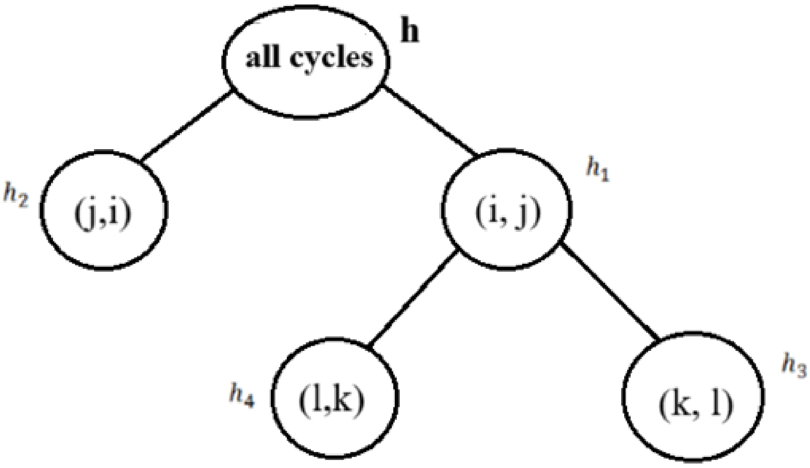

Figure 3.

Now we had to drop the Singapore row and Bali column of Table 5 and also prohibit the transition Bali → Singapore. But in the previous table, there is no Bali → Singapore line because we dropped the Bali line. Each line must have one pass. Therefore, we will prohibit the passage from Malaysia to Singapore (via the ∞ symbol). After these changes, table 6 is created.

	Uzbekistan	Thailand	Singapore
Uzbekistan	∞	0	47
Malaysia	0	0	∞
Thailand	0	∞	31

Table 6.

As can be seen from the table, the coefficient of this table is calculated as 31 and $h_3 = 1016 + 31 = 1047$.

After that, it is necessary to define a new pair to branch the circle on the right. As before, the degrees of zero elements in the table are calculated, which are listed in Table 7. Zero elements with the largest degree are found in (Uzbekistan, Thailand) and (Thailand, Singapore) pairs, and we choose one of them. For example, the pair (Uzbekistan, Thailand) should be removed. Then we get the graph in Figure 4. In this case, the number 16, which is the degree of $c_{UT} = 0$, is added to 1047, and the number 1063 is written next to the corresponding left circle of the graph in Figure 4.

	Uzbekistan	Thailand	Singapore
Uzbekistan	∞	$0^{(16)}$	16
Malaysia	$0^{(0)}$	$0^{(0)}$	∞
Thailand	$0^{(0)}$	∞	$0^{(16)}$

Table 7.

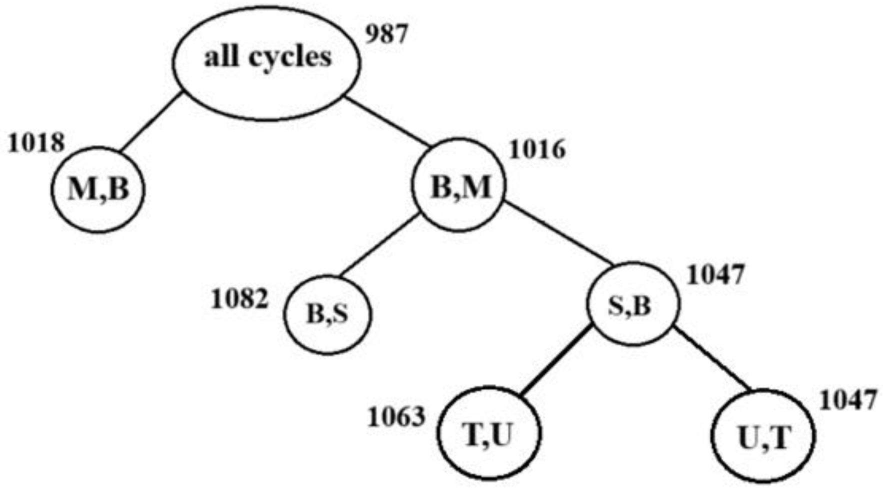

Figure 4.

The Uzbekistan row of the last 7th table, and the Thailand column are discarded and we prohibit the transition to Thailand → Uzbekistan (via the ∞ sign). Table 8 is created from these changes.

This table has a citation factor of 0. Therefore, the border corresponding to the right circle in Figure 4 has not changed (1047).

	Uzbekistan	Singapore
Malaysia	0	∞
Thailand	∞	0

Table 8.

Thus, we got the resulting graph (Fig. 4). Using the sequence of circles on the right side of this graph and the last $(2 \times 2) -$ size 8 table, the cycle Uzbekistan→Thailand→Singapore→Bali→Malaysia→Uzbekistan with a cost of 1047 is determined.

Summary

In this article, the issue of minimizing the total costs for trips between countries of the tourism firm "Fresh Tour" was considered. In this case, using the traveling salesman problem (TSP) method, the firm can find the optimal route covering different destinations. The article analyzed the mathematical model of this problem, algorithms and optimization methods. As a result, the tourism firm was able to effectively manage travel costs and maximize the use of resources.

References:

1. M. Tokhtasinov. Process Research. Textbook. 2017. -572 p.
2. D.L. Applegate, R.E. Bixby, V. Chvatal, W.J. Cook. “The Traveling Salesman Problem: A Computational Study”
3. N.P. Buslenko, V.V. Krinitsyn. S.Yu. Gorodesky. “Transportation Problems and Ways to Solve Them” Optimal Management. NNGU. 2014.

4. Z. Mamatova, M. Rasuljonova. “Creating a Mathematical Model for Finding the Optimal Solution of Total Costs for Traveling Between Countries for Tourism Firms Using the Traveling Salesman Problem” Collection of Proceedings of the International Scientific and Technical Conference on the Role of Digital Technologies in the Development of Exact and Natural Sciences: Problems and Innovative Solutions. Part 3. 2024. pp. 75-81.