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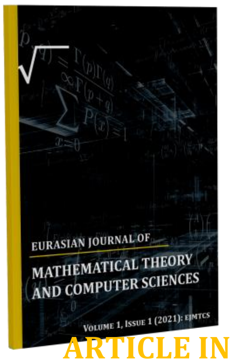

BA'ZI TRANSSENDENT FUNKSIYALARNING INTEGRALINI HISOBLASH USULLARI

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ABSTRACT

Ko'pchilikka ma'lum hozirda integrallash mavzusi nafaqat Oliy ta'lim muossasalarida balki umumiy o'rta ta'lim maktablarida ham o'qitish anchagina murakkab masalalardn biri bo'lib kelmoqda. Shu jumladan transsendent funksiyalarni integralini topish masalasi ham hozirda dolzarb masalalardan biri bo'lib hisoblanadi.

Kirish

Hozirgi kunda mamlakatimizda matematika sohasini rivojlantirishga qator e'tibor qaratilmoqda. Jumladan, yurtimizda matematika 2020-yildagi ilm-fanni rivojlantirishning ustuvor yo'nalishlaridan biri etib belgilandi. 2020-2021-o'quv yilidan boshlab Al-Xorazimiy nomidagi hududiy fan olimpiyadasini o'tkazish tartibi tasdiqlandi. Qoraqalpog'iston Respublikasi Nukus shaxrida joylashgan NDPI qoshidagi litsey negizida NDPIning matematikaga ixtisoslashtirilgan maktabi tashkil etildi. Shu qatorda shuni aytib o'tishimiz joizki, nafaqat matematikani balki boshqa fanlarni rivojlantirishda ham integrallarni chuqur o'rganish muhim masalalardan biri hisoblanadi.

Bularni hisobga olgan holda biz ba'zi transsendent funksiyalarning integrallarini hisoblash usullarini ko'rib chiqamiz.

Bilamizki, transsendent funksiyalarning bir nechta turlari mavjud. Bizga berilgan transsendent funksiya qaysi ko'rinishda berilishiga qarab ularning integrallash usullari ham funksiyalarga mos ravishda o'zgarib boradi. Bunda biz quyidagi hollarni ko'rib chiqamiz.

I. Biz tirigonametrik ko'rinishdagi transsendent funksiyalarni integrallashni asosan uchga bo'lib ko'rib o'tamiz:

I.1. $\int R(\sin x, \quad \cos x)dx$ ko'rinishidagi integral bo'lsa, uni $t = tg\frac{x}{2}$ almashtirish yordamida $\int R(\sin x, \quad \cos x)dx$ funksiya t ning ratsional funksiyasiga keltiriladi.

Misol 1. $\int \frac{dx}{5+3\sin x}$ integralni toping.

$$\int \frac{dx}{5+3\sin x} = \left| \begin{matrix} t = tg\dfrac{x}{2} & \sin x = \dfrac{1-tg^2\dfrac{x}{2}}{1+tg^2\dfrac{x}{2}} = \dfrac{1-t^2}{1+t^2} \\ x = 2arctgt & dx = \dfrac{2dt}{1+t^2} \end{matrix} \right| = 2\int \frac{dt}{\dfrac{5(1+t^2)+3(1-t^2)}{1+t^2}} \cdot \frac{1}{1+t^2} =$$

$$= 2\int \frac{dt}{8+2t^2} = \frac{1}{2} arctg\frac{t}{2} + c = \frac{1}{2} arctg\frac{\sin x}{2} + c.$$

I.2. $\int R(\sin^2 x, \ \cos^2 x)dx$ ko‘rinishida bo‘lsa, uni $t = tgx$ almashtirish orqali $\int R(\sin^2 x, \ \cos^2 x)dx$ funksiya t ning ratsional funksiyasiga keltiriladi.

Misol 2. Ushbu $\sqrt{1+x^2}$ funksiyaning integralini toping.

$$\int \sqrt{1+x^2}dx = \left| \begin{matrix} x = tgt \\ dx = \dfrac{dt}{\cos^2 t} \end{matrix} \right| = \int \frac{\sqrt{1+tg^2 t}}{\cos^2 t}dt = \int \frac{dt}{\cos^3 t} = \int \frac{\cos tdt}{\cos^4 t} = \int \frac{d\sin t}{(1-\sin^2 t)^2} =$$

$$= |\sin t = y| = \int \frac{dy}{(1-y^2)^2} = \int \frac{dy}{(1-y)^2(1+y)^2} = \frac{1}{4}\int \frac{dy}{(1-y)^2} + \frac{1}{4}\int \frac{dy}{(1-y)} + \frac{1}{4}\int \frac{dy}{(1+y)^2} +$$

$$+ \frac{1}{4}\int \frac{dy}{(1+y)} = \frac{1}{4}\frac{1}{1-y} - \frac{1}{4}\frac{1}{1+y} - \frac{1}{4}\ln(1-y) + \frac{1}{4}\ln(1+y) + c = \frac{1}{2}\frac{y}{1-y^2} + \frac{1}{4}\ln\left|\frac{1+y}{1-y}\right| + c =$$

$$= \frac{1}{2}\frac{\sin t}{1-\sin^2 t} + \frac{1}{4}\ln\left|\frac{1+\sin t}{1-\sin t}\right| + c = \frac{1}{2}\frac{\sin t}{\cos^2 t} + \frac{1}{2}\ln\sqrt{\frac{1+\sin t}{1-\sin t}} + c\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln\left|x+\sqrt{1+x^2}\right| + c.$$

I.3 $\int R(\sin^m x, \ \cos^n x)dx$ ko‘rinishida bo‘lsa, m va n ko‘rsatkichlaridan bittasi toq bo‘lsa, $t = \cos x$ yoki $t = \sin x$ almashtirish orqali t ning ratsional funksiyasiga keltiriladi.

Misol 3. $\int \frac{dx}{\sqrt{(a^2-x^2)^3}}$ integralni toping.

$$\int \frac{dx}{\sqrt{(a^2-x^2)^3}} = \left| \begin{matrix} x = a\sin t \\ dx = a\cos tdt \end{matrix} \right| = \int \frac{a\cos tdt}{\sqrt{(a^2-a^2\sin^2 t)^3}} = \int \frac{a\cos tdt}{a^3\cos^3 t} = \frac{1}{a^2}\int \frac{dt}{\cos^2 t} = \frac{1}{a^2}tgt + c =$$

$$= \frac{1}{a^2}\frac{\sin t}{\cos t} + c = \frac{1}{a^2}\frac{\sin t}{\sqrt{1-\sin^2 t}} + c = \frac{1}{a^2}\frac{x}{\sqrt{a^2-x^2}} + c.$$

Aytaylik bizga $P(x)$ ko‘phad va $\sin\alpha x$ yoki $\cos\alpha x$ funksiyalardan iborat bo‘lgan $\int P(x)\sin axdx$ va $\int P(x)\cos axdx$ integralar berilgan bo‘lsin. U holda bu funksiyalarning integrallari quyidagi formulalar yordamida funksiyalarni integralini hisoblashimiz mumkin:

$$\int P(x)\sin ax = -\frac{\cos ax}{a}\left(P(x) - \frac{P''(x)}{a^2} + \frac{P^{(4)}(x)}{a^4} - \ldots\right) + \frac{\sin ax}{a^2}\left(P'(x) - \frac{P^{(3)}(x)}{a^2} + \frac{P^{(5)}(x)}{a^4} + \ldots\right) + c,$$

$$\int P(x)\cos ax = \frac{\sin ax}{a}\left(P(x) - \frac{P''(x)}{a^2} + \frac{P^{(4)}(x)}{a^4} - \ldots\right) + \frac{\cos ax}{a^2}\left(P'(x) - \frac{P^{(3)}(x)}{a^2} + \frac{P^{(5)}(x)}{a^4} + \ldots\right) + c.$$

Misol 4. Quyidagi funksiyalarni integralini toping.

1) $x^5 \sin 5x$ 2) $\left(x^2+1\right)^2 \cos x$

1) $\int x^5 \sin 5x dx = -\frac{\cos 5x}{5}\left(x^5 - \frac{20x^3}{25} + \frac{120x}{5^4}\right) + \frac{\sin 5x}{25}\left(5x^4 - \frac{60x^2}{5^2} + \frac{120}{5^6}\right) + c =$

$= -\frac{\cos 5x}{5}\left(x^5 - \frac{4x^3}{5} + \frac{24x}{125}\right) + \frac{\sin 5x}{25}\left(5x^4 - \frac{12x^2}{5} + \frac{24}{5^5}\right) + c.$

2) $\int \left(x^2+1\right)^2 \cos x dx = \sin x\left(\left(x^2+1\right)^2 - (12x^2+4) + 24\right) + \cos x\left(\left(4x^3+4x\right) - 24x\right) + c =$

$= \sin x\left(x^4 - 10x^2 + 21\right) + \cos x\left(4x^3 - 20x\right) + c.$

II. Agar bizga berilgan funksiya quyidagi ko‘rinishda bo‘lsa,

$\int P(x) e^{\alpha x} dx$ ($P(x)$ - n- darajali ko‘phad),

holda biz bo‘laklab integrallash formulasidan foydalanamiz. Bu yerda

$$\begin{bmatrix} u = P(x), & dv = e^{\alpha x} dx \\ du = P'(x) dx, & v = \frac{1}{\alpha} e^{\alpha x} \end{bmatrix}$$

kabi belgilash kiritamiz va $\int u dv = uv - \int v du$ formulasidan foydalanib integralini hisoblaymiz. Bu yerda P(x) ko‘phad o‘zgarmas songa aylangunicha bo‘laklab integrallashni davom ettiramiz. Natijada berilgan funksiyaning integrali quyidagi ko‘rinishga keladi.

$$\int P(x) e^{\alpha x} = \left(\frac{P(x)}{\alpha} + \frac{P'(x)}{\alpha^2} + \frac{P''(x)}{\alpha^3} + \frac{P'''(x)}{\alpha^4} + \ldots + \frac{1}{\alpha^{n+1}}\right) e^{\alpha x} + c.$$

Agar qavsni ichini $Q(x)$ ko‘phad deb belgilasak, quyidagi sodda ko‘rinishni oladi.

$$\int P(x) e^{\alpha x} = Q(x) e^{\alpha x} + c.$$

Misol 5. Quyidagi $\left(x^2+3x+1\right)e^{2x}$ funksiyaning integralini toping.

$$\int (x^2+3x+1)e^{2x}dx = \left| \begin{matrix} u = x^2+3x+1 & dv = e^{2x}dx \\ du = (2x+3)dx & v = \frac{1}{2}e^{2x} \end{matrix} \right| = \frac{x^2+3x+1}{2}e^{2x} - \int \frac{2x+3}{2}e^{2x}dx =$$

$$= \frac{x^2+3x+1}{2}e^{2x} - \left| \begin{matrix} u = x + \frac{3}{2} & dv = e^{2x}dx \\ du = dx & v = \frac{1}{2}e^{2x} \end{matrix} \right| = \frac{x^2+3x+1}{2}e^{2x} - \frac{2x+3}{4}e^{2x} + \int \frac{1}{2}e^{2x}dx =$$

$$= \frac{x^2+3x+1}{2}e^{2x} - \frac{2x+3}{4}e^{2x} + \frac{1}{4}e^{2x} + c = \frac{x^2+2x}{2}e^{2x} + c.$$

Agar R(x) – maxraji haqiqiy ildizga ega bo‘lgan ratsional funksiya bo‘lsa, u holda ushbu $\int R(x) e^{ax} dx$ integral elementar funksiyalar va transsendent funkiyalar orqali ifodalanadi, ya’ni

$$\int \frac{e^{ax}}{x} dx = li(e^{ax}) + c, \qquad lix = \int \frac{dx}{\ln x}$$

ko'rinishida bo'ladi.

Misol 6. Quyidagi funksiyaning integralini toping.

$$\int\left(\frac{x-2}{x}\right)^2 e^x dx=\int\left(1-\frac{4}{x}+\frac{4}{x^2}\right)e^x dx=\int\left(e^x-4\frac{e^x}{x}+4\frac{e^x}{x^2}\right)dx=e^x-4\int\frac{e^x}{x}dx+4\int\left(\frac{e^x}{x^2}\right)dx=$$

$$=e^x-4li(x)+4\left|\begin{matrix} u=e^x & dv=\frac{1}{x^2}dx \\ du=e^x dx & v=-\frac{1}{x}\end{matrix}\right|=e^x-4li(e^x)-4\frac{e^x}{x}+4\int\frac{e^x}{x}=\frac{x-4}{x}e^x+c.$$

Misol 7. Quyidagi funksiyaning integralini toping.

$$\int\frac{e^{2x}}{x^2-3x+2}dx=\int\frac{e^{2x}}{(x-2)(x-1)}dx=\int\frac{e^{2x}}{(x-2)}dx-\int\frac{e^{2x}}{(x-1)}dx=\int\frac{e^4e^{2(x-2)}}{(x-2)}d(x-2)-$$

$$-\int\frac{e^2e^{2(x-1)}}{x-1}d(x-1)=e^4li\left(e^{2x-4}\right)-e^2li\left(e^{2x-2}\right)+c$$

Agar bizga berilgan integral $\int R(e^x)dx$ ko'rinishida bo'lsa, u holda biz $\left|t=e^x,\quad x=\ln t,\quad dx=\frac{dt}{t}\right|$ almashtirishni bajaramiz.

Misol 8. Quyidagi $f(x)=\frac{e^{2x}}{e^x+1}$ funksiyaning integralini toping.

$$\int\frac{e^{2x}dx}{e^x+1}=\left|\begin{matrix} t=e^x \\ x=\ln t \\ dx=\frac{dt}{t}\end{matrix}\right|=\int\frac{t^2}{t+1}\frac{dt}{t}=\int\frac{t}{t+1}dt=\int(1-\frac{1}{t+1})dt=t-\ln(t+1)+c=e^x-\ln\left(e^x+1\right)+c.$$

Agarda bizga berilgan integralda ham trigonametrik ham ko'rsatkichli funksiya berilgan bo'lsa, u holda biz bo'laklab integrallash fo'rmulasi va berilgan integralni belgilash yordamida funksiya integralini topamiz.

Misol 9. Quyidagi transsendent funsiyaning integralni toping.

$$\int e^{ax}\cos^2 bxdx=\int e^{ax}\left(\frac{1+\cos 2bx}{2}\right)dx=\frac{1}{2}\int e^{ax}dx+\frac{1}{2}\int e^{ax}\cos 2bxdx=f(x)+g(x)$$

$$g(x)=\frac{1}{2}\int e^{ax}\cos 2bxdx=\left|\begin{matrix} u=e^{ax} & dv=\cos 2bxdx \\ du=ae^{ax}dx & v=\frac{1}{2b}\sin 2bx\end{matrix}\right|=\frac{1}{2}\left[\frac{1}{2b}e^{ax}\sin 2bx-\frac{a}{2b}\int e^{ax}\sin 2bxdx\right]=$$

$$=\frac{1}{4b}e^{ax}\sin 2bx-\frac{a}{4b}\left|\begin{matrix} u=e^{ax} & dv=\sin 2bxdx \\ du=ae^{ax}dx & v=-\frac{1}{2b}\cos 2bx\end{matrix}\right|=\frac{1}{4b}e^{ax}\sin 2bx+\frac{a}{8b^2}e^{ax}\cos 2bx-\frac{a^2}{8b^2}\int e^{ax}\cos 2bxdx=$$

$$=\frac{1}{4b}e^{ax}\sin 2bx+\frac{a}{8b^2}e^{ax}\cos 2bx-\frac{a^2}{8b^2}g(x)=g(x),$$

$$\frac{a^2+8b^2}{8b^2}g(x)=\frac{1}{4b}e^{ax}\sin 2bx+\frac{a}{8b^2}e^{ax}\cos 2bx+c,$$

$$g(x)=\frac{2b}{a^2+8b^2}e^{ax}\sin 2bx+\frac{1}{a^2+8b^2}e^{ax}\cos 2bx+c,$$

$$f(x)=\frac{1}{2}\int e^{ax}dx=\frac{1}{2a}e^{ax}+c,$$

$$\int e^{ax}\cos 2bxdx=f(x)+g(x)=\frac{2b}{a^2+8b^2}e^{ax}\sin 2bx+\frac{1}{a^2+8b^2}e^{ax}\cos 2bx+\frac{1}{2a}e^{ax}+c.$$

Xulosa qilib aytadigan bo‘lsak, transsendent funksiyalarni integrallashning nostandart usullaridan foydalanilgan va transssendent funksiyalarning integralini hisoblashga doir bir nechta misollar ko‘rsatib o‘tilgan.

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