



## IKKI O'ZGARUVCHILI TENGSIZLIK KON'YUNKSIYASI VA DIZ'YUNKSIYASI

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### ABSTRACT

Ushbu maqolada ikki o'zgaruvchili tengsizliklar sistemasini yechishda predikatlar algebrasi formulalaridan foydalanishning ahamiyati ochib berilgan. Ushbu maqolada ko'rib chiqilgan misollardan o'quvchilarga teoremlarni isbotlash va ikki o'zgaruvchili tengsizliklar sistemasini yechish usullarini o'rgatishda, shuningdek kreativ qobiliyatli o'quvchilar bilan ishlashda va matematika fanidan to'garak mashg'ulotlarini tashkil qilishda foydalanish mumkin.

Predikatlar algebrasining tengsizliklar sistemasini yechishga tadbirlarini o'rganishda uning tengkuchli formulalarini bilish muhim hisoblanadi. Biz predikatlar algebrasining quyidagi teng kuchli formulalaridan foydalanamiz:

$$P(x, y) \wedge (S(x, y) \vee Q(x, y)) \equiv P(x, y) \wedge S(x, y) \vee P(x, y) \wedge Q(x, y) \quad (1)$$

$$P(x, y) \vee S(x, y) \wedge Q(x, y) \equiv (P(x, y) \vee S(x, y)) \wedge (P(x, y) \vee Q(x, y)) \quad (2)$$

$$\overline{P(x, y) \wedge S(x, y)} \equiv \overline{P(x, y)} \vee \overline{S(x, y)} \quad (3)$$

$$\overline{P(x, y) \vee S(x, y)} \equiv \overline{P(x, y)} \wedge \overline{S(x, y)} \quad (4)$$

$$(x, y) \Rightarrow (x, y) \equiv P(x, y) \vee S(x, y) \quad (5)$$

$$P(x, y) \Rightarrow S(x, y) \equiv \overline{S(x, y)} \Rightarrow \overline{P(x, y)} \quad (6)$$

$$P(x, y) \Leftrightarrow S(x, y) \equiv P(x, y) \wedge S(x, y) \vee \overline{P(x, y)} \wedge \overline{S(x, y)} \quad (7)$$

$$P(x, y) \Leftrightarrow S(x, y) \equiv (\overline{P(x, y)} \vee S(x, y)) \wedge (\overline{S(x, y)} \vee P(x, y)) \quad (8)$$

Tengsizliklar predikatlardan iborat bo'lgani uchun tengsizlikni yechish masalasi predikatning rostlik sohasini topish masalasiga keladi.  $\mathcal{R}$  haqiqiy sonlar to'plami,  $P(x, y)$  va  $S(x, y)$  lar  $\mathcal{R}^2$  to'plamda aniqlangan predikatlar bo'lsin. Bu predikatlarining rostlik sohasini mos ravishda  $E_p$  va  $E_s$  lar bilan,  $P(x, y)$  predikatning inkorini  $\overline{P(x, y)}$  bilan va  $\mathcal{R}^2 \setminus E_p$  to'plamni  $\overline{E}_p$  bilan belgilaymiz.  $\overline{P(x, y)}$ ,

$P(x, y) \vee S(x, y)$ ,  $P(x, y) \wedge S(x, y)$ ,  $P(x, y) \Rightarrow S(x, y)$  va  $P(x, y) \Leftrightarrow S(x, y)$

predikatlarining rostlik sohasini topishda quyidagi teoremlardan foydalanamiz.

1-teorema.  $E_{\overline{p}} = \overline{E}_p$ .



Isbot.  $E_{\bar{p}}$  to'plamning ixtiyoriy elementini  $\overline{E_p}$  to'plamga tegishli ekanligini va  $\overline{E_p}$  to'plamga tegishli ixtiyoriy elementning  $E_{\bar{p}}$  to'plamga ham tegishli bo'lishini ko'rsatamiz.  $(x, y) \in E_{\bar{p}}$  to'plamga tegishli ixtiyoriy element bo'lsin. U holda  $\bar{P}(x, y) = 1$  bo'ladi. Bundan  $P(x, y) = 0$  va  $(x, y) \notin E_p$  bo'lishi kelib chiqadi. Bundan va  $(x, y) \in \mathcal{R}^2$  ekanligidan  $(x, y) \in \mathcal{R}^2 \setminus E_p = \overline{E_p}$ , ya'ni  $(x, y) \in \overline{E_p}$  bo'lishi kelib chiqadi. Endi  $\overline{E_p}$  to'plamga tegishli ixtiyoriy elementni  $E_{\bar{p}}$  to'plamga ham tegishli bo'lishini ko'rsatamiz:

$(x, y) \in \overline{E_p} \Rightarrow (x, y) \notin E_p \Rightarrow P(x, y) = 0 \Rightarrow \bar{P}(x, y) = 1 \Rightarrow (x, y) \in E_{\bar{p}}$  teorema isbot bo'ldi.

2-teorema.  $E_{p \vee s} = E_p \cup E_s$ .

Isbot.

$$\begin{aligned} \forall (S(x, y) = 1) &\Rightarrow ((x, y) \in E_p) \vee ((x, y) \in E_s) \Rightarrow (x, y) \in E_p \cup E_s \\ \forall (x, y) \in E_p \cup E_s &\Rightarrow (x, y) \in E_p \vee (x, y) \in E_s \Rightarrow (P(x, y) = 1) \vee \\ (x, y) \in E_{p \vee s} &\Rightarrow (P(x, y) \vee S(x, y) = 1) \Rightarrow ((P(x, y) = 1) \vee \end{aligned}$$

s.

$\forall (S(x, y) = 1) \Rightarrow P(x, y) \vee S(x, y) = 1 \Rightarrow (x, y) \in E_{p \vee s}$ . Teorema isbot bo'ldi.

3-teorema.  $E_{p \wedge s} = E_p \cap E_s$ .

Isbot.  $(x, y) \in E_{p \wedge s}$  bo'lsin. Bundan,

$$\begin{aligned} P(x, y) \wedge S(x, y) = 1 &\Rightarrow (P(x, y) = 1) \wedge (S(x, y) = 1) \Rightarrow \\ \Rightarrow ((x, y) \in E_p) \wedge ((x, y) \in E_s) &\Rightarrow (x, y) \in E_p \cap E_s \end{aligned}$$

$$\begin{aligned} (x, y) \in E_p \cap E_s \text{ bo'lsin, } &\Rightarrow ((x, y) \in E_p) \wedge ((x, y) \in E_s) \Rightarrow \\ \Rightarrow (P(x, y) = 1) \wedge (S(x, y) = 1) &\Rightarrow P(x, y) \wedge S(x, y) = 1 \Rightarrow \Rightarrow (x, y) \in E_{p \wedge s} \end{aligned}$$

teorema isbot bo'ldi.

4-teorema.  $E_{p \Rightarrow s} = \overline{E_p} \cup E_s$

Isbot.  $(x, y) \in E_{p \Rightarrow s}$  to'plamga tegishli ixtiyoriy element bo'lsin. U holda  $(P(x, y) \Rightarrow S(x, y) = 1)$  bo'ladi. Bundan va  $P(x, y) \Rightarrow S(x, y) \equiv \bar{P}(x, y) \vee (x, y) \in E_s$  ekanligidan  $(P(x, y) \vee S(x, y) = 1) \Rightarrow (P(x, y) = 1) \vee (S(x, y) = 1) \Rightarrow ((x, y) \in E_{\bar{p}}) \vee ((x, y) \in E_s) \Rightarrow (x, y) \in \overline{E_p} \cup E_s$  kelib chiqadi.

Endi  $\overline{E_p} \cup E_s$  to'plamga tegishli ixtiyoriy elementning  $E_{p \Rightarrow s}$  to'plamga ham tegishli bo'lishini ko'rsatamiz:  $(x, y) \in \overline{E_p} \cup E_s$  bo'lsin. U holda  $((x, y) \in \overline{E_p}) \vee ((x, y) \in E_s)$  bo'ladi.

Bundan va  $E_p = E_{\bar{p}}$  tenglikdan,  $((x, y) \in E_{\bar{p}}) \vee ((x, y) \in E_s) \Rightarrow (P(x, y) = 1) \vee ((x, y) = 1) \Rightarrow$

$\Rightarrow (P(x, y) \vee S(x, y) = 1) \Rightarrow (P(x, y) \Rightarrow S(x, y) = 1) \Rightarrow (x, y) \in E_{p \Rightarrow s}$  kelib chiqadi. Teorema isbot bo'ldi.



5-teorema.  $E_{p \leftrightarrow s} = (E_p \cap E_s) \cup (\bar{E}_p \cap \bar{E}_s)$

Isbot.  $(x, y) - E_{p \leftrightarrow s}$  to'plamga tegishli ixtiyoriy element bo'lsin. U holda

$(x, y) \leftrightarrow (x, y) = 1$  bo'ladi. Bundan va 7-tengkuchlilikdan

$$\Rightarrow (x, y) \wedge (x, y) \vee P(x, y) \wedge S(x, y) = 1 \Rightarrow$$

$$\Rightarrow ((x, y) \wedge (x, y) = 1) \vee (P(x, y) \wedge S(x, y) = 1) \Rightarrow$$

$$\Rightarrow ((x, y) = 1) \wedge ((x, y) = 1) \vee (P(x, y) = 1) \wedge (S(x, y) = 1) \Rightarrow$$

$$\Rightarrow ((x, y) \in E_p) \wedge ((x, y) \in E_s) \vee ((x, y) \in E_{\bar{p}}) \wedge ((x, y) \in E_{\bar{s}}) \Rightarrow$$

$$\Rightarrow ((x, y) \in E_p \cap E_s) \vee ((x, y) \in E_{\bar{p}} \cap E_{\bar{s}}) \Rightarrow (x, y) \in E_p \cap E_s \cup E_{\bar{p}} \cap E_{\bar{s}}.$$

Endi  $(x, y) \in E_p \cap E_s \cup \bar{E}_p \cap \bar{E}_s$  bo'lsin.

$$\text{Bundan, } ((x, y) \in E_p \cap E_s) \vee ((x, y) \in \bar{E}_p \cap \bar{E}_s) \Rightarrow$$

$$\Rightarrow ((x, y) \in E_p) \wedge ((x, y) \in E_s) \vee ((x, y) \in \bar{E}_p) \wedge ((x, y) \in \bar{E}_s) \Rightarrow$$

$$\Rightarrow ((x, y) = 1) \wedge ((x, y) = 1) \vee ((x, y) \in E_{\bar{p}}) \wedge ((x, y) \in E_{\bar{s}}) \Rightarrow$$

$$\Rightarrow ((x, y) = 1) \wedge ((x, y) = 1) \vee (P(x, y) = 1) \wedge (S(x, y) = 1) \Rightarrow$$

$$\Rightarrow ((x, y) \wedge (x, y) = 1) \vee (P(x, y) \wedge S(x, y) = 1) \Rightarrow$$

$$\Rightarrow (x, y) \wedge (x, y) \vee P(x, y) \wedge S(x, y) = 1. \text{ Bundan va (7) dan}$$

$(x, y) \leftrightarrow (x, y) = 1 \Rightarrow (x, y) \in E_{p \leftrightarrow s}$  kelib chiqadi. Teorema isbot bo'ldi.

6-teorema.  $E_{p \leftrightarrow s} = (\bar{E}_p \cup E_s) \cap (\bar{E}_s \cup E_p)$ .

Bu teorema ham 5-teoremaga o'xshash isbotlanadi. Faqat bu holda (7)-teng kuchli formula o'rniga (8)-teng kuchli formuladan foydalanish kerak.

O'quvchilarga predikatlar algebrasining tengkuchli formulalaridan foydalanib tengsizliklar yechishni va teorema isbotlash usullarni o'rgatishda quyidagi teoremlardan foydalanish mumkin.

7-teorema.  $(\forall (x, y) \in R^2)(P(x, y) \Rightarrow S(x, y)) \Rightarrow (E_p \subset E_s)$ .

8-teorema.  $E_p \subset E_s \Rightarrow (\forall (x, y) \in R^2)(P(x, y) \Rightarrow S(x, y))$ .

9-teorema.  $(\forall (x, y) \in R^2)(P(x, y) \Leftrightarrow S(x, y)) \Rightarrow (E_p = E_s)$ .

10-teorema.  $(E_p = E_s) \Rightarrow (\forall (x, y) \in R^2)(P(x, y) \Leftrightarrow S(x, y))$  [1].

Bu teoremlar teskarisidan faraz qilish usuli bilan oson isbotlanadi. Biz isbotini keltirish bilan cheklanamiz.

10-teoremani

Isbot. Teskarisidan faraz qilish usulidan foydalanamiz.

$$\neg(\forall (x, y) \in R^2)(P(x, y) \Leftrightarrow S(x, y)) \equiv$$

$$(\exists (x, y) \in R^2)(P(x, y) \not\leftrightarrow S(x, y)) \equiv$$

$$\equiv (\exists (x, y) \in R^2)(P(x, y) \wedge \neg S(x, y) \vee \neg P(x, y) \wedge S(x, y)) \equiv$$

$$\equiv (\exists (x, y) \in R^2)((P(x, y) \vee S(x, y)) \wedge (P(x, y) \vee \neg S(x, y))) \equiv$$

$$\equiv (\exists (x, y) \in R^2)(P(x, y) \wedge S(x, y) \vee \neg S(x, y) \wedge P(x, y)). \text{ Bundan}$$

$$\exists (x, y) \in R^2((P(x, y) \wedge S(x, y) = 1) \vee (\neg S(x, y) \wedge P(x, y) = 1)) \equiv$$

$$(\exists (x, y) \in R^2)((P(x, y) = 1) \wedge (S(x, y) = 1) \vee (\neg S(x, y) = 1) \wedge (P(x, y) = 1)) \equiv$$

$$(\exists (x, y) \in R^2)((x, y) = 0) \wedge ((x, y) = 1) \vee ((x, y) = 0) \wedge (P(x, y) = 1) \equiv$$

$$(\exists (x, y) \in R^2)((\overline{(x, y) \in E_p}) \wedge ((x, y) \in E_s) \vee (\overline{(x, y) \in E_s}) \wedge ((x, y) \in E_p)) \Rightarrow$$

$\Rightarrow \bar{E}_p \neq E_s \Rightarrow E_p \neq E_s$ . Teorema isbot bo'ldi.

$$|x| + |y| < 2$$

1-misol.  $\{x^2 + y^2 < 4\}$  tengsizliklar sistemasini yeching.



Yechish. Berilgan tengsizliklar sistemasidagi birinchi tengsizlikni  $(x, y)$  bilan, ikkinchi tengsizlikni  $(x, y)$  bilan belgilaymiz. U holda berilgan masala  $(x, y) \wedge (x, y)$  predikatning rostlik sohasi  $(E_{p \wedge s})$  ni topish masalasiga keladi.  $(E_{p \wedge s})$  ni topish uchun predikatlar algebrasining yuqorida keltirilgan asosiy teng kuchli formulalaridan va 2-3-teoremlardan foydalanamiz.

$$(x, y) \equiv (|x| + |y| < 2) \equiv (x \geq 0) \wedge (x - 2 < y < -x + 2) \vee \\ \vee (x < 0) \wedge (-x - 2 < y < x + 2) \equiv (x \geq 0) \wedge (y > x - 2) \wedge (y < -x + 2) \vee \vee (x < 0) \wedge (y > -x - 2) \wedge \\ (y < x + 2).$$

Hosil bo'lgan predikatning rostlik sohasini koordinata tekisligida tasvirlaymiz.

$x \geq 0$  soha OY o'qining o'ng tomonidagi yarim tekislikdan iborat bo'lgani uchun, bu yarim tekislikda  $y > x - 2$  predikatning rostlik sohasi  $y = x - 2$  to'g'ri chiziqning yuqori qismidan,  $y < -x + 2$  predikatning rostlik sohasi  $y = -x + 2$  to'g'ri chiziqning quyi qismidan iborat bo'ladi.

Hosil qilingan sohalarning kesishmasi

$$(x \geq 0) \wedge (y > x - 2) \wedge (y < -x + 2)$$

predikatning rostlik sohasi bo'ladi. Shunga o'xshash OY o'qining chap tomonidan iborat sohada

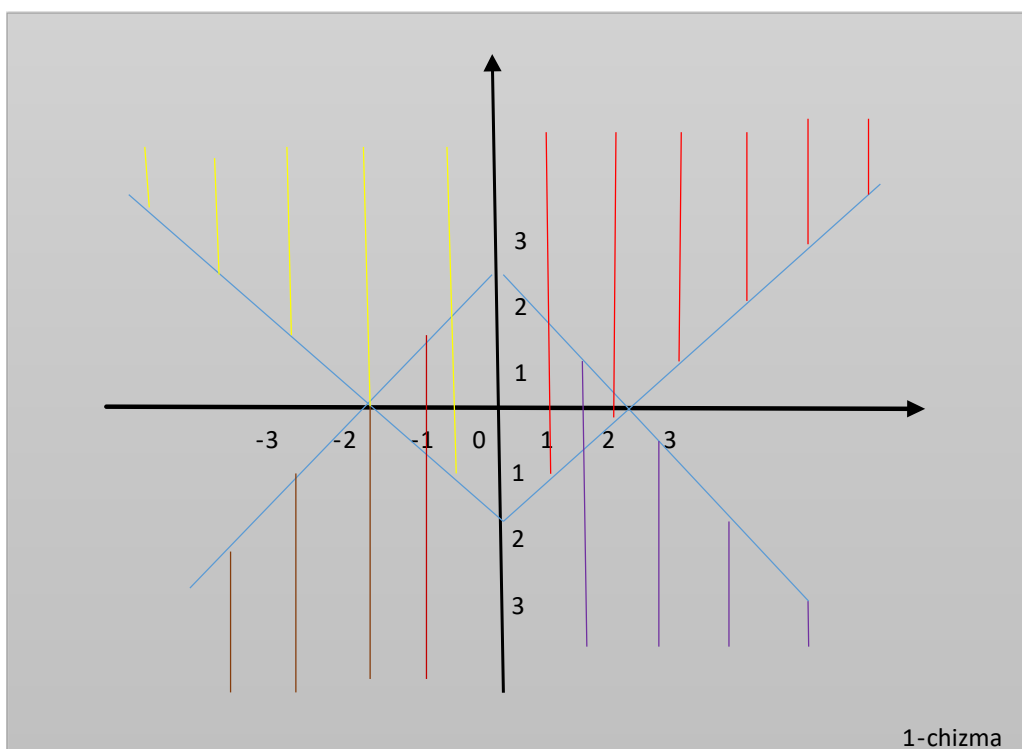
$$(x < 0) \wedge (y > -x - 2) \wedge (y < x + 2)$$

predikatning rostlik sohasini aniqlaymiz. Berilgan predikatning rostlik sohasi

$$(x \geq 0) \wedge (y > x - 2) \wedge (y < -x + 2) \text{ va}$$

$$(x < 0) \wedge (y > -x - 2) \wedge (y < x + 2)$$

predikatlarining aniqlangan rostlik sohalari birlashmasidan iborat bo'ladi. Koordinata tekisligida  $y > x - 2, y < -x + 2, y > -x - 2, y < x + 2$  predikatlarining aniqlangan rostlik sohalari shtrixlab borilsa, ikki (eng ko'p) marta shtrixlangan soha  $P(x, y)$  predikatning rostlik sohasi bo'ladi (1-chizma).



1-chizma



$(x, y) \Rightarrow (x^2 + y^2 < 4)$  predikatning rostlik sohasi  $E_s$  koordinata tekisligida markazi  $(0; 0)$  nuqtada va radiusi 2 ga teng bo'lgan aylananing ichki nuqtalari to'plamidan iborat, ya'ni bunday aylana ichida yotgan nuqtalarning koordinatalari

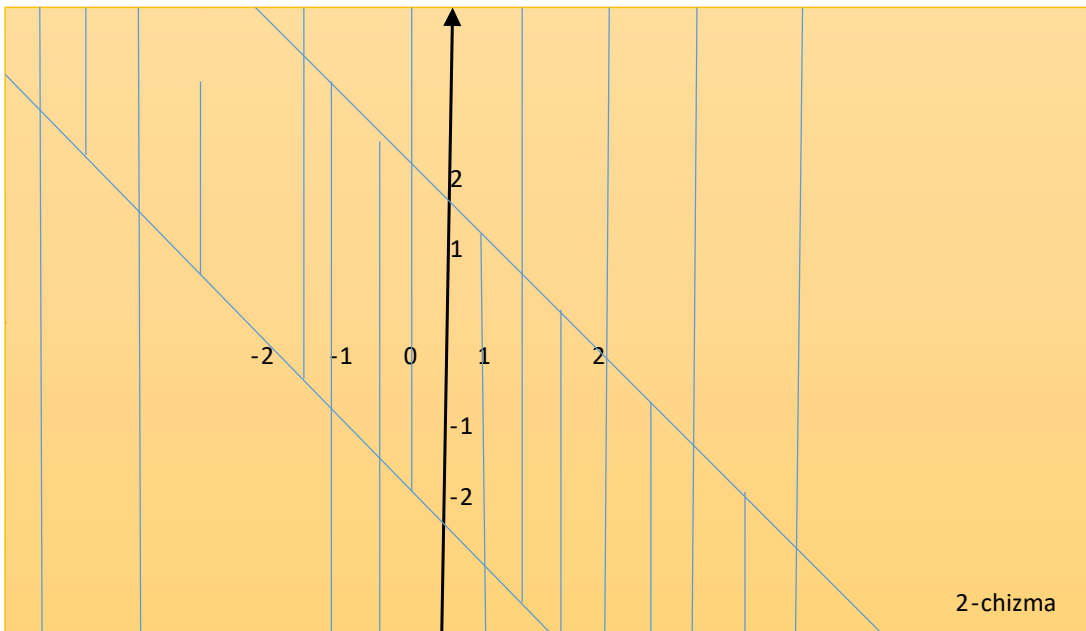
$x^2 + y^2 < 4$  predikatni chin mulohazaga (to'g'ri sonli tengsizlikka) aylantiradi. Bundan ko'rinadiki  $(x, y) \Rightarrow (|x| + |y| < 2)$  predikatning rostlik sohasi  $(x, y) \Rightarrow (x^2 + y^2 < 4)$  predikatning rostlik sohasining qism to'plami bo'lar ekan, ya'ni  $E_p \subset E_s$  ekan. Shuning uchun  $E_{p \wedge s} = E_p \cap E_s = E_p$   $E_{p \wedge s} = E_p \cap E_s = E_p$  bo'ladi.  $E_p \subset E_s$  munosabat va 11-teoremadan quyidagi  $(\forall(x, y) \in R^2)(|x| + |y| < 2 \Rightarrow x^2 + y^2 < 4)$  formulaning teorema ekanligi kelib chiqadi.

2-misol.  $|x + y| < 2$  tengsizlikni yeching.

Yechish. Berilgan tengsizlikni  $Q(x, y)$  bilan belgilaymiz. U holda berilgan masala  $Q(x, y)$  predikatning rostlik sohasi ( $E_q$ ) ni topish masalasiga keladi.

$$(x, y) \equiv (x + y > -2) \wedge (x + y < 2) \equiv (y > -x - 2) \wedge (y < -x + 2)$$

$Q(x, y)$  predikatning rostlik sohasi  $y = -x - 2$  va  $y = -x + 2$  parallel to'g'ri chiziqlar orasidagi sohadan iborat. 2-chizma.



2-chizma

$E_p \subset E_q$  munosabat va 11-teoremadan  $(\forall(x, y) \in R^2)(|x| + |y| < 2 \Rightarrow |x + y| < 2)$  formulaning teorema ekanligi kelib chiqadi.

3-misol.  $\begin{cases} |x - y| < 3 \\ |x + y| < 3 \end{cases}$  tengsizliklar sistemasini yeching.

4-misol. Quyidagi teoremlarni isbot qiling.

$$(\forall(x, y) \in R^2)(x^2 + y^2 < 4 \Rightarrow |x| + |y| < 2)$$

$$(\forall(x, y) \in R^2)(x^2 + y^2 < 2 \Rightarrow |x| + |y| < 2).$$

Yuqorida ko'rib chiqilgan misollardan o'quvchilarga predikatlar algebrasining tadbirlarini o'rgatishda foydalanish mumkin. O'quvchilarga matematik mantiq fani qonuniyatlari, keltirib chiqarish qoidalari, tengkuchli formulalari va ularning tadbirlari chuqur va atroflicha o'rgatib



borilsa, ularning matematik masalalarni eng sodda usullarda, tez va hatosiz yechish qobiliyatlari rivojlanib boradi.

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