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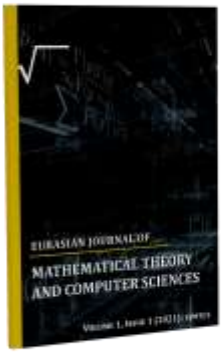

BISINGULYAR INTEGRALNING BA'ZI BIR XOSSALARI VA ULARNING TATBIQLARI

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ABSTRACT

Ushbu maqolada jamlanuvchi funksiyalar fazosida bisingulyar integral uchun Zigmund tipidagi tengsizlik olindi. Bu tengsizlik asosida integral operatorga nisbatan invariant banax fazosi quruldi. Keyin esa bu fazoda chiziqli bo'lmagan integral tenglamaning yechimining mavjud va yagonaligi isbot qilindi.

Quyidagi bisingulyar integralni qaraymiz.

$$\tilde{u}(x_1,x_2)=\int_{a_1}^{a_2}\int_{b_1}^{b_2}\frac{u(s_1,s_2)}{(s_1-x_1)(s_2-x_2)}ds_1ds_2,$$

bu yerda $u\in L_p^{loc}(\Delta)$, $\Delta=(a_1,a_2;b_1,b_2), p>1$

Quyidagi belgilashlarni kiritamiz .

$$L_p^{loc}(a_1,a_2,b_1)=\{u-o'lchovli|\forall\xi_1,\eta_{1,}\xi>0,\xi_1+\eta_1\le a_2-a_1=l_1,\xi\epsilon(0,l_2]\ l_2=b_2-b_1,u\in L_p[a_1+\xi_1,a_2-\eta_1,b_1+\xi,b_2]\}.$$

$$L_p^{loc}(a_1,a_2,b_2)=\{u-o'lchovli|\forall\xi_1,\eta_{1,}\xi>0,\xi_1+\eta_1\le a_2-a_1=l_1,\xi\epsilon(0,l_2]l_2=b_2-b_1,u\in L_p[a_1+\xi_1,a_2-\eta_1,b_1,b_2-\xi]\}.$$

Quyidagi funksiyalarni kiritamiz.

$$\Omega_{p,1}(u,\xi_1,\eta_1,\xi)=\left(\int_{a_1+\xi_1}^{a_2-\eta_1}\int_{b_1}^{b_2-\xi}|u(x_1,x_2)|^p dx_1dx_2\right)^{\frac{1}{p}},$$

$$\Omega_{p,2}(u,\xi_1,\eta_1,\xi)=\left(\int_{a_1+\xi_1}^{a_2-\eta_1}\int_{b_1}^{b_2-\xi}|u(x_1,x_2)|^p dx_1dx_2\right)^{\frac{1}{p}},$$

$$\omega_{p,1}(u,\delta,\xi_1,\eta_1,\xi)=\sup_{h\epsilon E}\left(\int_{a_1+\xi_1}^{a_2-\eta_1}\int_{b_1+\xi}^{b_2-h}|u(x_1,x_2+h)-u(x_1,x_2)|^p dx_1dx_2\right)^{\frac{1}{p}},$$

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$$\omega_{p,2}(u,\delta,\xi_1,\eta_1,\xi)=\sup_{h\epsilon E}\left(\int_{a_1+\xi_1}^{a_2-\eta_1}\int_{b_1}^{b_2-h-\xi}|u(x_1,x_2+h)-u(x_1,x_2)|^p dx_1dx_2\right)^{\frac{1}{p}}$$

bu yerda $p>1, E=\{h{:}\,0<h\le min\,\{\delta,l_2-\xi\}\}$.

2.1. – teorema.$u_i\epsilon\, L_p^{loc}(a_1,a_2,b_i)$ $p>1,\xi_i^i\,\epsilon(0,l_2], i=1,2$ bo'lsin . U holda mos integrallar integrallanuvchi bo'lganda quyidagi tengsizliklar o'rinli.

$$\Omega_{p,i}(\tilde{u}_i,\xi_1,\eta_1,\xi^i)\le$$

$$\le C\left[\frac{1}{\xi_1^{\frac{1}{q}}}\int_0^{\frac{\xi_1}{2}}\int_0^{\frac{l_2}{2}}\frac{\Omega_{p,i}\left(u_i,t_1,\frac{l_1}{2},t_2\right)}{t_1^{\frac{1}{p}}t_2^{\frac{1}{p}}(t_2+\xi^i)^{\frac{1}{q}}}dt_1dt_2+\frac{1}{\eta_1^{\frac{1}{q}}}\int_0^{\frac{\eta_1}{2}}\int_0^{\frac{l_2}{2}}\frac{\Omega_{p,i}\left(u_i,\frac{l_1}{2},t_1,t_2\right)}{(t_1t_2)^{\frac{1}{p}}(t_2+\xi^i)^{\frac{1}{q}}}dt_1dt_2\right.$$

$$+\frac{1}{\xi_1^{\frac{1}{q}}}\int_0^{\frac{\xi_1}{2}}\int_0^{\frac{\xi^i}{2}}\frac{\omega_{p,i}\left(u_i,t_1,\frac{l_1}{2},t_2,\frac{\xi^i}{2}\right)}{t_1^{\frac{1}{p}}t_2}dt_1dt_2+\frac{1}{\eta_1^{\frac{1}{q}}}\int_0^{\frac{\eta_1}{2}}\int_0^{\frac{\xi^i}{2}}\frac{\omega_{p,i}\left(u_i,\frac{l_1}{2},t_1,t_2,\frac{\xi^i}{2}\right)}{t_2t_1^{\frac{1}{p}}}dt_1dt_2$$

$$\left.+\beta(\xi^i)\Omega_{p,j}\left(u_j,\frac{l_1}{2},\frac{l_1}{2},\frac{l_2}{3}\right)\ln\frac{l_2}{\xi^i}\right],$$

bu yerda

$$\beta(\xi^i)=\begin{cases}1, x_2\epsilon\left(0,\frac{l_2}{3}\right]\\ 0, x_2 \bar{\in} \left(0,\frac{l_2}{3}\right]\end{cases}$$

Bisingulyar integral operator uchun invariant fazo.

$\bar{G}$ orqali mos ravishda $\{\xi_1,\eta_1,0<\xi\le l_2,\xi_1+\eta_1\le l_1\}$ va $\{0<\delta,\xi_1,\eta_1,\xi,\xi_1+\eta_1\le l_1\}$ da aniqlangan musbat $(\varphi(\xi_1,\eta_1,\xi),\psi(\delta,\xi_1,\eta_1,\xi))$ funksiyalar juftini belgilaymiz.

Bu funksiyalar ξ_1,η_1,ξ argumentlar bo'yicha kamayuvchi, $\psi(\delta,\xi_1,\eta_1,\xi)$ funksiya δ bo'yicha deyarli o'suvchi$\frac{\psi(\delta,\xi_1,\eta_1,\xi)}{\delta}$ deyarli kamayuvchi hamda $\delta\to 0$ da $\psi(\delta,\xi_1,\eta_1,\xi)\to 0$.

$(\varphi_1,\psi_1)\in\bar{G}$ bo'lsin. $H_{\varphi_1\psi_1}^{p,b_1}$ orqali $L_p^{loc}(a_1,a_2,b_1)$ dagi shunday jamlanuvchi u funksiyalarni to'plamini belgilaymizki,bunda

$$\exists C_1,C_2>0,\Omega_{p,1}(u,\xi_1,\eta_1,\xi)\le C_1\varphi_1(\xi_1,\eta_1,\xi)$$
$$\omega_{p,1}(u,\delta,\xi_1,\eta_1,\xi)\le C_2\psi(\delta,\xi_1,\eta_1,\xi)$$

shartlar bajarilsin.

$H_{\varphi_1\psi_1}^{p,b_1}$ fazoda normani quyidagicha kiritamiz:

$$\|u\|_{H_{\varphi_1\psi_1}^{p,b_1}}=max\left\{\sup_{\xi_1,\eta_1,\xi}\frac{\Omega_{p,1}(u,\xi_1,\eta_1,\xi)}{\varphi(\xi_1,\eta_1,\xi)},\ \sup_{\delta,\xi_1,\eta_1,\xi}\frac{\omega_{p,1}(u,\delta,\xi_1,\eta_1,\xi)}{\psi(\delta,\xi_1,\eta_1,\xi)}\right\}$$

$H_{\varphi_1\psi_1}^{p,b_1}$ fazo bu norma bo'yicha banax fazosi bo'ladi.

X_1va X_2 lar biror chiziqli X fazodagi Banax fazolari bo'lsin . X_1va X_2larning arifmetik yig'indisini X_1+X_2 orqali belgilaymiz ,ya'ni

$$X_1+X_2=\{x\in X|\exists x_1\in X_1,\exists x_2\in X_2, x=x_1+x_2\}$$

X_1+X_2 da normani quyidagicha kiritamiz .

$\|x\|_{X_1+X_2}=\inf_{\{x_1\in X_1, x_2\in X_2, x=x_1+x_2\}} max\{\|x_1\|_{X_1},\|x_2\|_{X_2}\}$

Bu norma bo'yicha X_1+X_2fazo Banax fazosi bo'ladi.

Ta'rif X_1,X_2,Y_1 ,Y_2 lar Banax fazolari , M,N chiziqli fazolar ,X_1,$X_2\subset M$,Y_1 ,$Y_2\subset N$ va Y_1^0, Y_2^0 lar N dagi Banax fazolari bo'lsin .

Agar $AX_1\subset Y_1\ +Y_2^0$, $AX_2\subset Y_2\ +Y_1^0$ va A:$X_1\to Y_1\ +Y_2^0$ chegaralangan , A:$X_2\to Y_2\ +Y_1^0$ chegaralangan bo'lsa , A chiziqli operator X_1+X_2 ni Y_1+Y_2 ga ko'chirib ,

(Y_1^0, Y_2^0) juft bo'yicha chegaralangan deyiladi. (Y_1^0, Y_2^0)bo'yicha chegaralanganlik,

A operatorning X_1+X_2 ni Y_1+Y_2 ga ko'chirib , chegaralanganlik tushunchasi bilan bir xildir .

$\bar{G}_0$ orqali $\bar{G}$ dagi shunday (φ,ψ) funksiyalar to'plamini olamizki ,$\forall\xi\in(0,l_2]$,$\forall\xi_1$,$\eta_1\in(0,l_1]$ da quyidagi integrallar yaqinlashuvchi bo'lsin .

$$\int_0^{l_1}\int_0^{l_2}\frac{\varphi\left(t_1,\frac{l_1}{2},t_2\right)}{(t_1t_2)^{\frac{1}{p}}(t_2+\xi)^{\frac{1}{q}}}dt_1dt_2\,,\int_0^{l_1}\int_0^{l_2}\frac{\varphi\left(\frac{l_1}{2},t_1,t_2\right)}{(t_1t_2)^{\frac{1}{p}}(t_2+\xi)^{\frac{1}{q}}}dt_1dt_2$$

$$\int_0^{l_1}\int_0^{l_2}\frac{\psi\left(t_1,\frac{l_1}{2},t_2,\frac{\xi}{2}\right)}{t_1^{\frac{1}{p}}t_2}dt_1dt_2\ ,\int_0^{l_1}\int_0^{l_2}\frac{\psi\left(\frac{l_1}{2},t_1,t_2,\frac{\xi}{2}\right)}{t_1^{\frac{1}{p}}t_2}dt_1dt_2$$

Quyidagi belgilashlarni kiritamiz :

$H^{p,b_1}_{\ln\frac{2l_2}{\xi^1},\frac{\delta}{\delta+\xi^1}}=M_1^{0,p}$,$H^{p,b_2}_{\ln\frac{2l_2}{\xi^2},\frac{\delta}{\delta+\xi^2}}=M_2^{0,p}$

$\bar{G}_0H_p$ orqali $\bar{G}_0$dagi shunday funksiyalar to'plamini belgilaymizki ,quyidagi shartlar bajarilsin.

$$1.\int_0^{\xi_1}\int_0^{l_2}\frac{\varphi(t_1,t_2)}{(t_1t_2)^{\frac{1}{p}}(t_2+\xi)}dt_1dt_2=O\big(\varphi(\xi_1,\xi)\big)$$

$$2.\int_0^{\xi_1}\int_0^{\xi}\frac{\psi\left(t_1,t_2,\frac{\xi}{2}\right)}{t_1^{\frac{1}{p}}t_2(t_2+\delta)}dt_1dt_2=O\big(\psi(\xi_1,\delta,\xi)\big)$$

$$3.\ \psi(\xi_1,\delta,\xi)=O\big(\varphi(\xi_1,\xi)\big)$$

$$4.\frac{\delta}{\delta+\xi}\ \varphi(\xi_1,\xi)=\ O\big(\psi(\xi_1,\delta,\xi)\big)$$

2.2. – teorema. $(\varphi_1,\psi_1),(\varphi_2,\psi_2)\in\bar{G}_0H_p$ bo'lsin. U holda $\tilde{u}$ operator ni $H^{p,b_1}_{\varphi_1\psi_1}+H^{p,b_2}_{\varphi_2\psi_2}$ o'zini – o'ziga o'tkazadi va$\left(M_1^{0,p},M_2^{0,p}\right)$ bo'yicha chegaralangan bo'ladi.

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