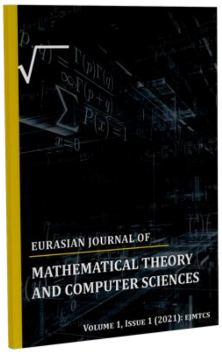
EURASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES
Innovative Academy Research Support Center
IF = 7.906 www.in-academy.uz



STOCHASTIC PROGRAMMING UNDER PROBABILISTIC CONSTRAINTS AND ITS APPLICATION IN AUTHENTICATION SYSTEMS

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https://doi.org/10.5281/zenodo.14645622

ARTICLE INFO
Received: 08th January 2025
Accepted: 13th January 2025
Online: 14th January 2025
KEYWORDS
Stochastic programming, multi-factor authentication, cybersecurity, probability maximization.

ABSTRACT

Stochastic programming has emerged lately as an advanced mathematical method to develop a decision-making process under uncertainty. This work investigates its use in enhancing Multi-Factor Authentication systems by establishing a probability- maximized framework for dynamic authentication processes in real time. Traditional MFA methods, though effective in reducing the risks of compromised credentials, usually have problems with user friction, lack of adaptability, and are vulnerable against advanced threats, including phishing and social engineer- ing. We propose a dynamic approach where stochastic models are integrated into MFA to adjust the authentication requirements based on real-time risk evaluations and user behavior for better security and user experience.

Dynamic risk evaluation models, flexible authentication mechanisms, and con- tinuous adaptation strategies will be pursued in the methodology to mitigate the challenges associated with traditional MFA systems. Case studies in the fi- nancial sector give meaning to the proposed framework by demonstrating how adaptive authentication measures may be aligned with transaction risk. For in- stance, high-risk scenarios utilize robust mechanisms such as biometric scans or hardware tokens, while low-risk situations use simpler methods like passwords with SMS codes. These results demonstrate a drastic reduction in the likelihood of unauthorized access by the proposed probability-maximized MFA framework compared to traditional 2FA systems. For instance, in a high-risk compromise, the probability reduced from 0.0005 to 0.000001, hence enhancing the security and scalability of the developed model. This interdisciplinary application bridges the gap from stochastic programming theory to cybersecurity practice and sets the scene for further research into adaptive authentication systems, which bal- ance robust security with convenience of user interaction.

1 Introduction

Stochastic programming are a powerful math approach for solving optimization problems where uncertainty exist. It help decision-makers deal with random and unpredictable situations by using probability concepts, which make their results more reliable and adaptable.

As cyber attacks gets more complex and sophisti- cated, we needs new ways to protect sensitive systems and data. Multi-factor authentication (MFA) have become crucial in modern cybersecurity, using sev- eral verification steps for confirming if users are legitimate. This research paper look at combining stochastic programming with MFA to create new method for optimizing authentication dynamically based on real-time risk assessment. The goal are to improve security and user experience by adapting the authentication process for each specific login attempt. Unlike traditional static MFA systems that doesn't account for evolving cyber threats, this new approach offer more flexibil- ity. The paper begins by discussing two-factor authentication (2FA), which is key part of MFA and how it protect user accounts. While 2FA helps reduce risks when passwords is compromised, it also face challenges like compatibility issues and being vulnerable to phishing and social engineering attack. The research then examine the limitations of current MFA systems and show how stochastic pro- gramming could address these problem. The main contribution of this study is developing a stochastic model that maximize the chances of successful authentica- tion in MFA systems. The proposed framework combine dynamic risk assessment, flexible authentication methods, and continuous adaptation strategies to provide comprehensive response to constantly changing cybersecurity threats. A financial sector case study demonstrates how probability-based approaches can enhance authentication processes based on changing risk level. This paper bridges the gap between theoretical developments in stochastic programming and its practi- cal applications in cybersecurity field.

2 Understanding 2FA/MFA Systems

Two-Factor Authentication (2FA), which requires users to give two different forms of authentication, greatly improves the security of online accounts. Usually, two distinct types of authentication elements are merged in this way: either a code transmitted to a mobile device (possession-based) or a password (knowledge- based) coupled with a biometric marker (inherence-based). This multi-layered approach mitigates the risks associated with compromised passwords and strength- ens the overall security of user accounts Das et al. (2020).

2.1 Key Authentication Factors

Authentication mechanisms for 2FA or MFA are broadly classified into the follow- ing categories:

- Knowledge-Based /actors: These include elements that the user knows, such as passwords, PINs, or answers to security questions. Research shows that despite being one of the oldest methods, knowledge-based factors remain vulnerable to brute-force attacks and phishing Bonneau et al. (2012a).
- Possession-Based /actors: These involve items the user possesses, such as hardware tokens, one-time-password (OTP) generators, or smartphones used to receive SMS-based or app-generated codes. Although highly effec- tive, they can be susceptible to theft or interception Tiwari et al. (2021).
- Biometric /actors: These rely on unique physiological or behavioral char- acteristics of the user, such as fingerprints, facial recognition, or voice pat- terns. Biometric-based systems have gained popularity due to their conve- nience, but concerns about privacy and spoofing vulnerabilities persist Ratha et al. (2001).

- Location and Time /actors: Authentication based on the user's geographic location or the timing of access attempts is increasingly employed, partic- ularly in contexts where adaptive security mechanisms are required Chiou et al. (2022).

2.2 Challenges and Considerations in 2FA/MFA Implementation

While 2FA and MFA (Multi-Factor Authentication) significantly enhance security, they also introduce certain challenges that must be addressed:

- Increased User Complexity: Users may find multi-factor systems incon- venient due to the additional steps required during the authentication pro- cess Weir et al. (2009).
- Deployment Costs: Organizations may face significant costs for implement- ing and maintaining MFA systems, especially in environments with legacy infrastructure Chiasson et al. (2007).
- Compatibility with Existing Systems: Ensuring seamless integration with legacy systems remains a technical challenge Maqousi et al. (2022).
- Vulnerabilities to Phishing and Social Engineering: Despite the added layers, advanced phishing tactics and social engineering attacks can still bypass certain MFA implementations Acar et al. (2016).

2.3 Variants of MFA Implementations

Below are some notable variants of MFA systems, widely deployed across indus- tries:

- Push Notification Authentication: Sends a notification to the user's mo- bile app for approval.
- Time-Based One-Time Passwords (TOTP): OTPs that expire after a short duration and are generated by an app like Google Authenticator.
- Biometric Authentication: Increasingly adopted in mobile and desktop environments, such as fingerprints or face id.
- Adaptive M/A: Incorporates contextual information such as the user's lo- cation, device, or behavior to adjust the required authentication factors dynamically.

These variants cater to different organizational needs and strike a balance be- tween user convenience and security. The growing adoption of 2FA/MFA highlights the critical role of these systems in securing sensitive data in sectors like financial technology, healthcare, and cloud computing Johnson et al. (2020).

3 Related Work

Despite its potential to address the shortcomings of static authentication tech- niques, the use of stochastic programming in cybersecurity, specifically in improv- ing Multi-Factor Authentication (MFA), is still largely unexplored. The current state of research on MFA systems, their difficulties, and the growing application of stochastic approaches in the field of cybersecurity are highlighted in this sec- tion.

3.1 Research on Multi-Factor Authentication Systems

MFA systems are widely recognized for their ability to enhance security by em- ploying multiple layers of authentication, including knowledge-based (e.g., pass- words), possession-based (e.g., hardware tokens), and inherence-based (e.g., bio- metrics) factors. These systems, however, face notable challenges:

- User /riction and Adaptability: Traditional MFA methods often inconve- nience users due to static, rigid protocols that fail to account for varying levels of risk. Research by Bonneau et al. Bonneau et al. (2012b) identifies user friction as a significant barrier to MFA adoption, emphasizing the need for more dynamic and context-aware systems.

• Vulnerabilities to Advanced Threats: Even robust MFA systems are sus- ceptible to phishing, social engineering, and man-in-the-middle attacks Burt & et al. (2014). These threats highlight the need for dynamic models that can adapt authentication requirements in real time based on evolving risks.

3.2 Stochastic Programming in Cybersecurity

Stochastic programming, a method for making decisions when things are uncer- tain, could be a valuable tool to overcome the shortcomings of MFA. This ap- proach has already proven successful in areas like managing supply chains and energy systems, showing its ability to optimize operations in the face of unpre- dictable events. However, its use in cybersecurity is still relatively new.

• Dynamic Risk Evaluation: Stochastic programming's ability to incorpo- rate probabilistic risk evaluations makes it well-suited for adaptive MFA systems. For example, Birge and Louveaux Birge & Louveaux (2011) demon- strate how stochastic methods can optimize decision-making by modeling uncertainty, a concept that can be directly applied to dynamic authentica- tion.

• Cybersecurity-Specific Applications: Limited work exists on the applica- tion of stochastic models in authentication. Research by Zhang et al. Zhang & et al. (2021) explores probabilistic frameworks for access control, but their models lack the flexibility to address real-time adaptation in MFA.

• Limitations in Current Approaches: While advancements in probabilistic methods for cybersecurity have been made, such as distributionally robust optimization Gao & Kleywegt (2016), they are rarely tailored to address user convenience and real-time adaptability, both critical to MFA systems.

4 Implementation of MFA Methods

4.1 SMS and Email Codes

Common but less secure; these methods involve sending codes to a user's phone or email. They are susceptible to interception, phishing, and SIM-swapping attacks.

4.2 Authentication Apps

These apps generate time-limited codes, providing greater security than tradi- tional SMS. Examples include Google Authenticator and Microsoft Authenticator.

4.3 Hardware Tokens

Devices like YubiKey generate one-time passwords or utilize public-key cryptog- raphy for secure authentication.

4.4 Biometric Authentication

This method uses unique physical characteristics, such as fingerprints, facial recog- nition, or iris scans, to verify users. It is harder to spoof but may raise concerns about privacy and biometric data security.

5 Probability Maximization Benefits in MFA Implementation

Multi-Factor Authentication (MFA) aims to achieve a balance between strong se- curity and user convenience. Its core function is to verify user identities with maximum accuracy while effectively preventing unauthorized access. By inte- grating probabilistic models, MFA systems can become more adaptive. These models enable the authentication process to dynamically adjust based on real- time risk evaluations and the unique behavior patterns of individual users.

Traditional two-factor authentication (2FA) systems, however, operate within a fixed framework. They typically require users to authenticate using a predefined set of methods, such as passwords, which can be less flexible and potentially less secure. In contrast, probability maximization models dynamically evaluate real-time factors to improve security effectiveness and user convenience. The probability maximization problem for MFA can be formulated as:

$$\max P\left(T_a > \xi\right) \quad (1)$$

Subject to:

$$A_x = b_x \geq 0 \qquad (2)$$

Where:

- T_a represents the authentication outcome threshold.
- ξ is the security threshold parameter.
- *A* and *b* denote system constraints. Prekopa (1995)

For maximizing the probability of correct authentication across multiple fac- tors, the following general constraint-based formulation can be applied:

$$\max P\left(\text{auth}_1(x,\epsilon) \geq 0, \text{auth}_2(x,\epsilon) \geq 0, \ldots, \text{auth}_r(x,\epsilon) \geq 0\right) \qquad (3)$$

Subject to:

$$x \in \mathrm{D} \quad (4)$$

Prekopa (1995) Here:

- authi(x, ϵ) represents the authentication outcome for factor i with system state x and noise ϵ.
- D represents the feasible domain of input parameters.

This approach ensures that the combined probability of successful authentica- tion across all factors is maximized.

6 Strategies for Maximizing Probability in Implementation

To implement stochastic programming models in MFA effectively, the following strategies should be considered:

6.1 Data Gathering and Evaluation

- Importance: Ongoing observation of user behavior, access trends, and de- vice usage is crucial to develop accurate baseline data.
- Application: Statistical models such as Markov Chains or Bayesian Net- works can be employed to predict user behavior and detect anomalies in real-time.

6.2 Dynamic Risk Evaluation Models

- Description: Dynamic models assess the security threat of each login at- tempt in real-time and adjust authentication requirements accordingly.
- Implementation: Leverage stochastic programming to evaluate risks using parameters such as device integrity, location accuracy, login time, and ha- bitual usage patterns. For example, a model might assign higher weights to unusual locations or unfamiliar devices.

- Mathematical /ormulation:

$$R(x) = \sum_{i=1}^{n} w_i f_i(x) \qquad (5)$$

Where:

– R(x) is the risk score for the login attempt.

– w_i are weights for each factor $f_i(x)$ based on its importance.

6.3 Flexible Authentication Methods

- Approach: Based on the risk level, the system adapts the authentication technique dynamically. For instance:

– Low-risk attempts may require only a password.

– High-risk attempts may demand biometric verification or hardware tokens.

- Example: A stochastic threshold ϑ can define risk levels:

if $R(x) > \vartheta$, then additional factors are required. (6)

6.4 Continuous Improvement and Adaptation

- Rationale: Security models and protocols must adapt to counter new be- havior patterns and emerging threats.
- Solution: Machine learning algorithms such as reinforcement learning can continuously optimize authentication models by learning from new data.

6.5 Integration with Stochastic Programming

- Stochastic Constraints: Multi-factor systems can implement constraints like:

$$P\left(A_i(x, \epsilon) \geq \gamma_i\right) \geq \alpha \qquad (7)$$

Where:

renewcommand

– Ai(x, ϵ) is the authentication factor's output.

– γi is the factor-specific threshold.

– α is the acceptable probability level.Gupta (2023)

By integrating stochastic programming methods, MFA systems can achieve higher security while maintaining usability, dynamically adapting to real-time risks.

7 Strategies for Maximizing Probability in Implementation

To incorporate this method into two-factor authentication systems, we need to take into considerations a few essential steps:

1. Data Gathering and Evaluation: Ongoing observation of user behaviors and access trends is crucial for developing accurate baseline data.
2. Dynamic Risk Evaluation Models: These models assess the security threat of each login attempt in real time and adjust the authentication require- ments as needed Awati.
3. /lexible Authentication Methods: Based on the risk level of each login attempt, the system modifies its authentication techniques immediately.
4. Continuous Improvement and Adaptation: It is vital that the security models and protocols adapt continuously to counter new patterns of behav- ior and potential threats Citrix.

8 Case Study on using MFA methods in financial sector

Let consider Case Study that we can combine above methods in real life. The fi- nancial sector, which is acutely susceptible to information breaches, would make a good example. In this case, MFA for the banking sector or the example of proba- bility maximization may be the perfect example for future application: the banks may implement the system where the probability of the event determines the au- thentication process. More specifically, the process may vary depending on the transaction risk level. For instance: 1. A password and an SMS code for low-risk;

2. A biometric scan and a hardware token for high-risk, etc. Consequently, the system may adjust how many stages or types of process should take place based on the real-time risk evaluation. This would allow financial institutions to maxi- mize the event likelihood as well as to ensure the best possible security for their users.

/urther Case study Implementation in the Financial Sector

To illustrate further example, let's take a bank already implemented a dynamic MFA system. It should monitor user behavior actions, transaction history, and device integrity to build a risk profile for each user. When a user initiates a transaction, the system determines the risk level based on these factors.

Step 1: Risk Assessment: The risk, according to the user's behavior, the amount of the transaction, and the safety level of the device, is assessed. For example, a high-risk score would be required to log in from an unfamiliar device.

Step 2: Dynamic Authentication: The level of required authentication dynam- ically changes based on what the result of the risk scoring will be: low risk will require basic authentication (password + SMS code), and a high-risk transac- tion will require more security (biometric + hardware token).

Step 3: Continuous monitoring: The system continuously learns new data, re- fining the risk models to better detect and respond to threats. :

8.1 Probabilities of tradition password comprimise and Probability maximised MFA

In usual two-factor authentication (2FA) is when we use two different ways to prove who we are. In this, we say: a password is an SMS code sent to your phone. And this gives us the following chances of something serious happening(Let's assume below probablities for simplicity):

There's a 5 % chance that somebody could guess our password (0.05). There's a 1 % chance that somebody could catch our SMS code (0.01). :

Anyhow, now let's consider that there is no way we can know the probability for both a password and an SMS code to be stolen simultaneously. We multiply these probabilities: Chance of both being hacked = 0.05 × 0.01 = 0.0005. The probablity of both passwords being hacked can be thus written as 0.05 %.

Scenario	Traditional 2/A	Proposed M/A
High-Risk Login	0.0005	0.000001
Low-Risk Login	0.02	0.001

Table 1: Comparison of Traditional 2FA and Proposed MFA

8.2 Probability-Maximization MFA

A more challenging system of this type is multi-factor authentication (MFA), in which case the system adapts to how risky a situation seems. Let's consider two situations: low-risk and high-risk. We have seen probability measure of low-risk scenario above example. Now let's continue from high-risk case.

8.3 High-Risk Scenario

In a high-risk scenario, the system uses more robust methods such as:

Biometric scan, like a fingerprint or hardware token Physical device that we own.

Suppose there's a 0.1 probability that someone might steal or spoof our biomet- ric scan, that is 0.001. The probability that someone steals our hardware token

= 0.1 = 0.001

Probability of both these methods being compromised: Probability of both = 0.001 × 0.001 = 0.000001. Therefore, the probability that both a biometric scan and a hardware token are compromised at the same time is 0.0001.

9 Discussion

In terms of making multi-factor authentication (MFA) systems better, our research focuses to integrate stochastic programming method for improving security while keeping good usability. Stochastic programming handle uncertainty by changing processes dynamically based on real-time assessment, which make it valuable ad- dition to cybersecurity framework. Traditional two-factor authentication (2FA) systems depends on static methods, such as passwords and SMS codes, for veri- fying users identity. Even though they is effective at reducing risks from stolen credentials, these fixed approaches has limitations, including more user friction and being vulnerable for phishing or social engineering attacks. In comparison, a probability-maximized MFA system use flexible and adaptive authentication tech- niques. It utilize probabilistic models to dynamically adjusts the authentication level based on risk assessments, how users behave, and context factors like if the device is secure or where its located. Our study investigates how this approach can makes breaches less likely in high-risk scenarios without making the user ex- perience worse when risks is low. For instance, high-risk scenarios might need strong authentication factors such as biometric scans or hardware tokens, while low-risk transactions could use more basic methods like passwords together with SMS codes. The case study in financial sector shows how this adaptive model works in practice, demonstrating its ability for matching security measures with how risky transactions are. The results shows that a probability-maximized MFA system significant reduces the chances of unauthorized access happening. When you compare it to regular 2FA systems, which has a fixed compromise probability of 0.0005, the adaptive MFA system gets a much lower compromise probability of 0.000001 when risks are high. This better security is especially important in industries like finance, where there's lots at stake and preventing fraud matter most. By constantly watching user behavior, making risk models better, and changing authentication protocols, probability-maximized MFA provide a scalable and ef- fective answer to modern cybersecurity challenges. It helps bridge the gap among strict security measures and smooth user experiences, making sure there is higher protection against new threats that emerge.

10 Conclusions

This study demonstrates the potential for stochastic programing to transform Multi-Factor Authentication (MFA) systems in significant ways. By examining the fundamental limitations of conventional MFA approaches - including user in- convenience, rigid protocols, and weaknesses against sophisticated attacks - this research puts forward a probability-maximized framework that actively adjusts authentication requirements according to real-time risk assessments. Through the implementation of stochastic models, it enables adaptive security measures that successfully balance strong protection with user convenience, ultimately deliver- ing an improved authentication experience. The system leverages dynamic risk assessment models, flexible authentication methodologies, and continuous adap- tation strategies to substantially decrease the probability of unauthorized system access. Real-world case studies conducted within the financial sector demon- strate the practical effectiveness of this approach, showing how it successfully aligns authentication mechanisms with transaction risk levels. In high-risk sce- narios specifically, the implementation of biometric scanning and hardware tokens resulted in a dramatic reduction in compromise probability - dropping from 0.0005 in traditional two-factor authentication systems to 0.000001 within the new proposed framework. This research effectively bridges the divide between theoretical stochastic programming and its practical cybersecurity applications, offering a scalable and robust solution capable of addressing modern security threats. Through its ongoing refinement of risk models and ability to adapt au- thentication protocols in response to emerging challenges, this interdisciplinary methodology not only enhances security but maintains strong usability, establish- ing a solid foundation that will support future developments in adaptive authentication systems.

11 Further questions in the topic

1. How we determine probability of dynamically choosing correct authentica- tion in more complex situation?
2. How can stochastic programming help other than the financial industry when integrated into MFA systems?
3. What problems would potentially be when deploying dynamic authentica- tion systems in large-scale organizations?
4. How best can user privacy be maintained so that data will still be continu- ously gathered for evaluation of risk in MFA systems?

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