



## DIFFERENTIAL EQUATIONS OF HEAT AND MASS TRANSFER IN THE DRYING PROCESS OF RICE GRAIN

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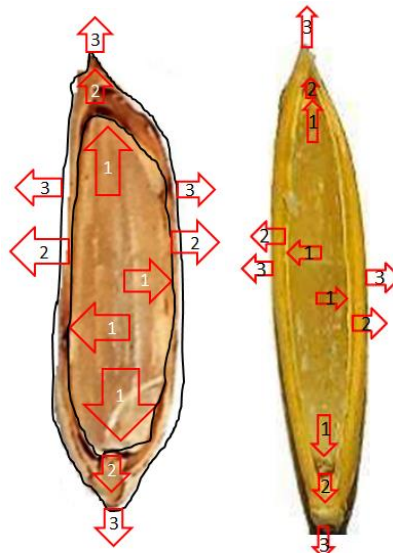
*Structure of rice grain, coefficients and gradients of heat and moisture transfer*

### ABSTRACT

*The article describes the peculiarities of rice grain, analyzes differential equations of heat and moisture transfer, and develops recommendations for calculating heat and moisture transfer during drying of rice grain.*

### INTRODUCTION

The unique structure of the rice grain requires many features to be taken into account in calculating its heat and moisture. According to the structure of the rice grain, there is a rice, a husk, and an air layer between them (Figure 1). When rice is dried convectively, the heat provided by the heat transfer agent passes through the husk to the air layer, and then reaches the rice. To bring the rice grain to the storage moisture content, the moisture content must be brought to 12-13 percent. To do this, the previously described heat and moisture transfer process must be carried out in reverse order. Now the amount of heat supplied to the rice grain must be increased or the amount of heat supplied must be maintained. This situation leads to a sharp increase in energy and time consumption.





1-heat and moisture transfer gradient in the rice; 2-heat and moisture transfer gradient in the air layer; 3-heat and moisture transfer gradient in the shell

Figure 1. Rice grain structure, heat and moisture transfer gradients

If we take into account the differences in heat and moisture transfer of the aforementioned organizers, it is not difficult to imagine that this process would take a long time.

During the drying process In the equation for heat transfer in capillary pores, the convective component is replaced  $c_{ij}\nabla T$  by the conductive component  $div(\lambda\nabla T)$  several times smaller than.

This result can be reached based on the following analysis. The total  $(Gr \cdot Pr) < 1 \cdot 10^3$  heat transfer coefficient in a dispersed (two or more phase) medium is equal to the molecular heat transfer coefficient, or heat transfer occurs by thermal conductivity.  $(Gr \cdot Pr) < 1 \cdot 10^3$  The value  $Re_{\exists} = 22$  corresponds to the equivalent criterion. We estimate the value of  $S h$ . When drying is carried out in convective dryers, the moisture transfer rate is approximately  $40 \frac{kg}{m^2 \cdot s}$  equal to. Under the most favorable conditions, the equivalent diameter of the capillary  $d_{\exists}$  is = 3 mm, or the porosity of the body is 70%. Viscosity of water at a temperature of  $30^{\circ} C$   $\eta_2 = 2.88 \frac{kg}{m \cdot s}$  is equal to. Then the criterion is equal to,

$$Re_{\exists} = \frac{j_2 d_{\exists}}{\eta_2 P} = \frac{40 \cdot 3 \cdot 10^{-3}}{0.7 \cdot 2.88} \approx 5.2 \cdot 10^{-2},$$

or several times smaller than 22. For this reason  $\sum_i c_{ij}\nabla T$ , it  $div(\lambda\nabla T)$  can be ignored when considering.

## DISCUSSION AND METHODS

Due to the absence of an overall pressure gradient, the system of differential equations for heat and moisture transfer takes the following form (Figure 1) :

heat and moisture transfer

$$\frac{\partial u}{\partial \tau} = k_{11}\nabla^2 u + k_{12}\nabla^2 T \tag{1}$$

$$\frac{\partial T}{\partial \tau} = k_{22}\nabla^2 T + k_{21}\nabla^2 u \tag{2}$$

this is it odds is defined as follows  $k_{11} = a_m, k_{12} = a_m^T = a_m \cdot \delta, k_{22} = a + a_{m1}^T \cdot \frac{r_{12}}{c}, k_{21} = a_{m1} \cdot \frac{r_{12}}{c}$

(1) and (2) is more generalized and is valid not only for the drying processes of wet materials, but also for arbitrary forms of heat and mass transfer.

The drying process is a typical heat and mass transfer process in motion. The source of moisture in it  $I_2 = -I_1 \frac{\partial u}{\partial \tau}$  can be expressed by the local humidity at time.

The total change in body moisture  $d_e u$  is equal to  $du$ , the change in moisture transfer and  $d_i u$  occurs due to the conversion of liquid into vapor, or,

$$\partial u = d_e u + d_i u. \tag{3}$$

In this case, body moisture is equal to the specific gravity of the fluid in the body ( $u = u_1 + u_2 = u_2$ ), or in the system under consideration  $i = 1, 2, u_1 = 0$ .

If the moisture transfer process is in motion ( $du \neq 0$ ), then  $du_i/du$  the ratio is the final value that determines the relative change in humidity due to evaporation of moisture at a given



point on the body. This value is called the phase change coefficient of the transformation of the liquid into vapor and is defined as:

$$\varepsilon = \frac{d_i u}{du} \tag{4}$$

If  $d_i u = 0$ , then the coefficient  $\varepsilon = 0$ , or indicates that moisture transfer occurs only by liquid transfer; if there is no liquid transfer ( $d_e u = 0$ ), or if the change in moisture at an arbitrary point of the body occurs by evaporation, the coefficient is equal to one. Thus, in the general case, the coefficient varies from zero to one, ( $0 \leq \varepsilon \leq 1$ ) and the differential equations for heat and mass transfer take the following form:

$$\frac{\partial u}{\partial \tau} = a_{m2} \nabla^2 u + a_{m2} \delta \nabla^2 T + \varepsilon \frac{\partial u}{\partial \tau} \tag{5}$$

$$\frac{\partial u}{\partial \tau} = a \nabla^2 T + \varepsilon \frac{r_{21}}{c} \frac{\partial u}{\partial \tau} \tag{6}$$

Equation (5) can be reduced to the following form by equating it with (6):

$$\frac{\partial u}{\partial \tau} = \frac{a_{m2}}{(1-\varepsilon)} [\nabla^2 u + \delta_2 \nabla^2 T], \tag{7}$$

with the equation

$$\frac{\partial u}{\partial \tau} = a_m (\nabla^2 u + \delta \nabla^2 T), \tag{8}$$

and in the implementation of the following :

$$a_m = \frac{a_{m2}}{1-\varepsilon} \quad \text{va} \quad \delta = \delta_2 \tag{9}$$

is correct for the most general case.

From here the phase shift coefficient takes the following form

$$\varepsilon = \frac{a_{m1}}{a_{m1} + a_{m2}} = \frac{a_{m1}}{a_m} \tag{10}$$

The system of equations for heat and moisture transfer (5) and (6) or (6) and (8) can be written in the form of the following system of equations (1) and (2), but in this case  $k_{ij} (i = 1,2; j = 1,2)$  the coefficients will be equal to:

$$k_{11} = a_{m2}(1 - \varepsilon) = a_m; \quad k_{12} = \frac{a_{m2} \delta_2}{1-\varepsilon} = a_m \delta; \tag{11}$$

$$k_{22} = a + \varepsilon \frac{r_{21}}{c} \frac{a_{m2} \delta_2}{1-\varepsilon} = a + \varepsilon \frac{r_{21}}{c} a_m \delta; \tag{12}$$

$$k_{21} = \varepsilon \frac{r_{21}}{c} \frac{a_{m2}}{1-\varepsilon} = \varepsilon \frac{r_{21}}{c} a_m$$

Thus , the source of vaporous moisture in the drying process in motion  $I_1$  is expressed by the following ratio

$$I_2 = -I_1 = \varepsilon \rho_0 \frac{\partial u}{\partial \tau}. \tag{13}$$

In that case,  $\varepsilon$  instead of the evaporation coefficient, the ice coefficient  $\varepsilon_3$  is obtained.

(13)  $\varepsilon$  allows us to express the ratio coefficient of the liquid flow with  $|j_2|$  the vapor flow  $|j_1|$ . In practice,  $\rho_0 \frac{\partial u}{\partial \tau}$  we substitute the following expressions

$$\rho_0 \frac{\partial u}{\partial \tau} = -div j_1 - div j_2$$

and the following we use equality

$$I_2 = -I_1 = \varepsilon \rho_0 \frac{\partial u}{\partial \tau}$$

and we have the following expression:

$$I_2 = \varepsilon \rho_0 \frac{\partial u}{\partial \tau} = -\varepsilon(\operatorname{div} j_1 + \operatorname{div} j_2) = -\operatorname{div} j_1 \quad (14)$$

from this

$$\varepsilon = \frac{\operatorname{div} j_1}{\operatorname{div} j_1 + \operatorname{div} j_2} = (1 + \frac{\operatorname{div} j_2}{\operatorname{div} j_1})^{-1} \quad (15)$$

Vectors in one-dimensional tasks  $j_1$  va  $j_2$  parallel or opposite direction, then :

$$\frac{\operatorname{div} j_2}{\operatorname{div} j_1} = \frac{(\frac{\partial}{\partial x})j_2}{(\frac{\partial}{\partial x})j_1} \quad (16)$$

If  $\varepsilon = \text{const}$  we assume that, then, naturally, the ratio (16)  $\frac{|j_2|}{|j_1|} = \text{const}$  must be constant and equal to, from formula (15) we obtain:

$$\varepsilon = \frac{|1|}{|1| + |j_2|} \quad (17)$$

For drying wet materials, the ratio (15) can be written as:

$$\varepsilon = \frac{a_{m1}(\nabla^2 u + \delta_1 \nabla^2 T)}{a_m(\nabla^2 u + \delta \nabla^2 T)} \quad (18)$$

we take the formula (9) as a polynomial  $\delta = \delta_1 = \delta_2$ , then  $\varepsilon = \frac{a_{m1}}{a_m}$  we obtain the equality, which was previously derived above.

Thus, to derive the coefficient of heat and moisture transfer in the process of drying in motion, it is necessary to follow the equation (9) or (17) when describing the liquid and vapor flows. These equations are satisfied in the hygroscopic state of the wet material. Deriving the coefficient using the equation (4) does not require a number of requirements to be met, including constancy with respect to the coordinate.

Formula (17) is used as a basis for deriving differential equations for heat and moisture transfer [L. 38].

From relation (17) the following expression follows

$$|j_1| = \frac{\varepsilon}{1-\varepsilon} |j_2|. \quad (19)$$

As a result, we get the following expressions:

$$j_1 = I_{n1}|j_1|; \quad j_2 = I_{n2}|j_2| \quad (20)$$

in  $I_{n1}$  va  $I_{n2}$  these n - separate vectors,  $j_1$  i  $j_2$  directed along the vectors.

Then from the differential equation

$$\rho_0 \frac{\partial u}{\partial \tau} = -\operatorname{div} j_1 - \operatorname{div} j_2 \quad (21)$$

we get:

$$\rho_0 \frac{\partial u}{\partial \tau} = -\operatorname{div} I_{n1}|j_1| - \operatorname{div} j_2 = -\operatorname{div} \frac{\varepsilon}{1-\varepsilon} |j_2| I_{n1} - \operatorname{div} j_2$$

If  $I_{n1} = I_{n2}$  we accept that,  $I_{n1}$  va  $I_{n2}$  indicates that it is directed in one direction, then we have :

$$\rho_0 \frac{\partial u}{\partial \tau} = -\operatorname{div} \frac{\varepsilon}{1-\varepsilon} j_2 - \operatorname{div} j_2 \quad (22)$$

Then, assuming that the coefficient ( $\varepsilon = \text{const}$ ) does not depend on the coordinate, we obtain from (22):

$$\rho_0 \frac{\partial u}{\partial \tau} = -\operatorname{div} j_2 + \varepsilon \rho_0 \frac{\partial u}{\partial \tau} \quad (23)$$



this n we define the expression for the moisture source:

$$I_2 = \varepsilon \rho_0 \frac{\partial u}{\partial \tau} \quad (24)$$

$I_{n1}$  va  $I_{n2}$  equality of vectors  $I_{n1} = I_{n2}$  is applied to the drying process, then the vectors are defined by the following expressions,

$$j_1 = -a_{m1}(\nabla u + \delta_1 \nabla T) \rho_0; \quad (25)$$

$$j_1 = -a_{m2}(\nabla u + \delta_2 \nabla T) \rho_0 \quad (26)$$

and have the same direction, for this to happen  $\delta_1 = \delta_2$ . This equality must be  $\delta_1 = \delta_2$  takes place in the hygroscopic field .

is valid for moving heat and moisture transfer when  $\frac{\partial u}{\partial \tau} \neq 0$ . For stationary heat and moisture transfer  $\varepsilon = \frac{d_i u}{du} \rightarrow \infty$ , because  $du = 0$ . Therefore, the value of the source  $I_2 = \infty \cdot 0$ , there is uncertainty.

To solve for the uncertainty, we use equation (5), from which we obtain:

$$\varepsilon \frac{\partial u}{\partial \tau} = \frac{\partial u}{\partial \tau} - a_{m2} \delta_2 \nabla^2 u - a_{m2} \delta_2 \nabla^2 T \quad (27)$$

In the stationary case  $\frac{\partial u}{\partial \tau} = 0$ , it follows that,

$$\varepsilon \frac{\partial u}{\partial \tau} = -a_{m2}(\nabla^2 u + \delta_2 \nabla^2 T) = \frac{1}{\rho_0} \text{div} j_2 \quad (28)$$

On the other hand, from equation (21)  $\frac{\partial u}{\partial \tau} = 0$ , we get :

$$\text{div} j_2 = -\text{div} j_1 \quad (29)$$

Thus , we obtain the expression for the heat source in steady flow

$$I_2 = \varepsilon \rho_0 \frac{\partial u}{\partial \tau} = \text{div} j_2 = -\text{div} j_1 \quad (30)$$

or the expression (15).

Naturally, the equation for the moisture source is

$$I_2 = -\text{div} j_1 = \text{div} \rho_0 (a_{m1} \nabla u + a_{m1}^T \nabla T) \quad (31)$$

It is more general in moisture and heat transfer for moving and stationary states in moist bodies.

Thus, the system of differential equations (1) and (2) remains the same, only the coefficient  $k_{ij}$  is determined by formulas (11) and (12).

In addition, the coefficient  $\varepsilon$  describes the mode of moisture in motion and heat transfer during the heating or cooling phase .

Heating intensity value

$$m = -\left(\frac{1}{T_c - T}\right) \frac{\partial T}{\partial \tau}. \quad (32)$$

$m$  The value is the last value and is equal to

$$m = \frac{a}{R_v^2} Bi \psi. \quad (32)$$

where  $n$  is  $\psi$  the unevenness of the temperature field , from 0 to 1 ( $0 \leq \psi \leq 1$ ),  $R_v$ — hydraulic radius of the body ,  $Bi$ — Bio criterion . In a stationary state  $\frac{\partial T}{\partial \tau} = 0$ , and the value is  $\left(\frac{1}{T_c - T}\right) = \infty$  ( $T = T_c$ ). Naturally,  $\varepsilon$  the heating rate, like the coefficient, is a characteristic of the



heat transfer in motion. However, the value of  $m$  does not mean that the heat transfer in motion varies from  $0 \infty$  to.

## RESULTS AND CONCLUSIONS

It should be noted that the following coefficients are used to calculate the temperature and humidity of the moving body during the drying process:  $a, a_m, a_m^T, \varepsilon$  and the thermodynamic  $c$  and  $r$  descriptions must be known. However, if the more general ratio (31) is used, then the thermophysical description is as follows  $a, c, a_{m1}, a, a_m, a_m^T, a_{m2}, a_m^T$  serves. Thus,  $\varepsilon$  introducing the coefficient reduces the number of parameters from 7 to 6. The system of differential equations for moisture and heat transfer (1) and (2) remains the same in all cases, only the expression of the coefficients when solving this system  $k_{ij}$  has different values depending on the thermophysical characteristics.

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