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A MATHEMATICAL MODEL FOR THE NUMERICAL SIMULATION OF OPTIMAL PARAMETERS FOR THE SPRING SUSPENSION OF HIGH-SPEED ELECTRIC TRAINS

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ABSTRACT

The article presents a mathematical model for numerical simulation of rational parameters of spring suspension of high-speed electric trains of the AFROSIAB type in the MATHCAD 15 programming environment.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДЛЯ ЧИСЛЕННОГО МОДЕЛИРОВАНИЯ РАЦИОНАЛЬНЫХ ПАРАМЕТРОВ РЕССОРНОГО ПОДВЕШИВАНИЯ ВЫСОКОСКОРОСТНЫХ ЭЛЕКТРОПОЕЗДОВ

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Электроподвижной состав, высокоскоростные электропоезда, моторный вагон,

ABSTRACT

В статье представлена математическая модель для численного моделирования рациональных параметров рессорного подвешивания высокоскоростных электропоездов типа АФРОСИАБ в среде программирования MATHCAD 15.

пружинная подвеска высокоскоростных электропоездов, численные исследования процесса колебаний, алгоритм, программа, среда программирования MATHCAD 15.

Efforts are underway globally to enhance the speed and capacity of railways through research and development. The focus is on improving the dynamic properties of high-speed electric trains by ensuring the appropriate selection and stability of hydraulic and hydro-friction vibration dampers, as well as optimizing spring suspension parameters. Enhancing the dynamic properties of electric rolling stock, improving the interaction forces between the wheel and the rail, ensuring wheel stability on the rail, and enhancing the smoothness of high-speed electric transport are crucial objectives. Simultaneously, a pressing task involves developing new designs and enhancing existing spring suspension systems for electric rolling stock, along with devising methods to calculate their dynamic strength. Addressing this challenge is vital to ensuring traffic safety and passenger comfort [1, 2].

Ultimately, the history of the development of the mechanical part of high-speed electric trains has led to the fact that the currently existing high-speed rolling stock, as a rule, has bogies with two stages of spring suspension, each of which contains elastic and dissipative elements. In the current state of the railway track, to ensure good smooth running of high-speed electric trains, passenger cars and locomotives, it is necessary to have a sufficiently “soft” spring suspension. For this purpose, pneumatic spring suspension is often used. Designs with such suspension make it easier to ensure the standard value of static deflection of 200 mm for passenger cars and rolling stock designed for speeds of 250 km/h and more. In addition, such suspension has both elastic and dissipative properties, i.e. no special damper is required [1,2].

Research has been conducted and is being conducted on this topic by leading scientists worldwide such as S.A. Brebbia (Wessex Institute of Technology, UK), G.M. Carlomagno (University of Naples di Napoli, Italy), A. Varvani-Farahani (Ryerson University, Canada), S.K. Chakrabarti (USA), S. Hernandez (University of La Coruna, Spain), S.-H. Nishida (Saga University, Japan). Authoritative scientific schools and prominent scientists in the CIS countries from MIIT, PGUPS, MAI, VNIIZhT, JSC VNIKTI, JSC Russian Railways, etc. have worked on these issues. A significant contribution to solving many complex problems and checking theoretical conclusions related to the study of the oscillation processes of the spring suspension of the rolling stock was made by the Russian Research Institute of Railway Transport (CNII MPS) and the

Russian Research Institute of Railcar Building (NIIV); there, along with theoretical studies, a large number of experimental studies (bench and full-scale ones) were conducted. In Uzbekistan, the academician of the Academy of Sciences of the Republic of Uzbekistan, Professor, Doctor of Technical Sciences Glushchenko A.D., Professors Fayzibaev Sh.S., Khromova G.A., Shermukhamedov A.A., D.O. Radjibayev and their students dealt with the problems of optimizing the systems of spring suspension of rolling stock [3÷7].

From the analysis of the above publications [1,2,4] concerning the study of oscillations using the method of mathematical modeling, it is clear that when creating a “crew–track” dynamic model, it is necessary to perform several stages:

- to determine the required number of degrees of freedom of the dynamic model, ensuring sufficient accuracy of the solution to the problem;

- to select a model of a railway track, adequately describing the vibrations of a railway track when a rolling stock moves along it;

- to select a method for checking the correctness of the compiled differential equations;

- to select a method for solving the resulting system of differential equations;

- to select the type of disturbances (deterministic or random) and the methods for modeling and specifying it in solving the problem;

- to select a method for processing the research results to determine the dependencies of the indices of the dynamic qualities of the rolling stock on its speed.

Figure 1 shows a spatial kinematic diagram of vertical vibrations of the car model of the high-speed electric train AFROSIAB. The car body rests on two two-axle bogies through the central stage of the spring suspension, and each of the bogies through the axle box stage on two wheelsets; springs and hydraulic dampers installed in parallel to them are used in the central stage of the spring suspension.

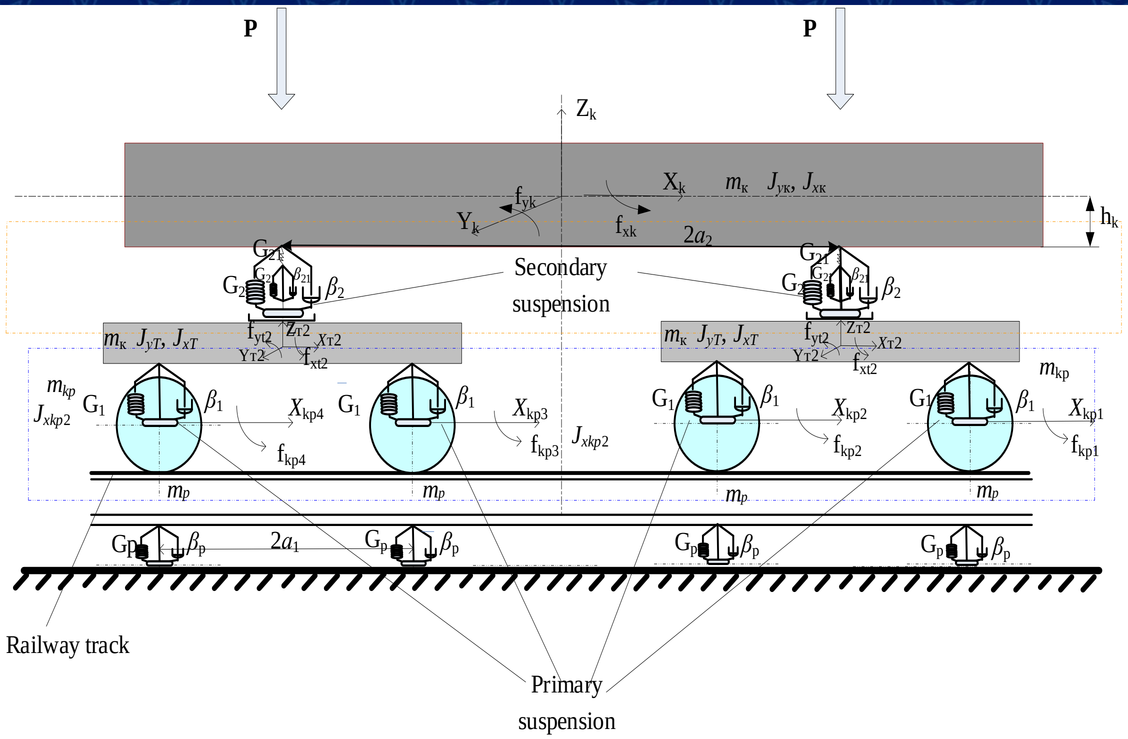

Figure 1. Spatial kinematic diagram of vertical oscillations of the car model of the high-speed electric train AFROSIAB.

The system of coupled differential equations describing forced vertical oscillations of the car model of the high-speed electric train AFROSIAB in the form of a five-mass system has the following form [5,6,7]

$$m_k\ddot{z}_k + 4\beta_2\dot{z}_k + 4G_2 z_k - 2\beta_2(\dot{z}_{т1} + \dot{z}_{т2}) - 2G_2(z_{т1} + z_{т2}) = 0;$$
$$J_{yk}\ddot{\varphi}_{yk} + 4\beta_2 a_2^2\dot{\varphi}_{yk} + 4G_2 a_2^2\varphi_{yk} + 2\beta_2 a_2(\dot{z}_{т1} - \dot{z}_{т2}) + 2G_2 a_2(z_{т1} - z_{т2}) = 0\,;$$
$$m_{т1}\ddot{z}_{т1} + (4\beta_1 + 2\beta_2)\dot{z}_{т1} + (4G_1 + 2G_2)z_{т1} - 2\beta_2\dot{z}_k - 2G_2 z_k + 2\beta_2 a_2\dot{\varphi}_{yk} + +2G_2 a_2\varphi_{yk}$$
$$\pm 2\beta_1(\dot{z}_{кр1} + \dot{z}_{кр2}) - 2G_1(z_{кр1} + z_{кр2}) = 0;$$
$$m_{т2}\ddot{z}_{т2} + (4\beta_1 + 2\beta_2)\dot{z}_{т2} + (4G_1 + 2G_2)z_{т2} - 2\beta_2\dot{z}_k - 2G_2 z_k - 2\beta_2 a_2\dot{\varphi}_{yk} -$$
$$-2G_2 a_2\varphi_{yk} \pm 2\beta_1(\dot{z}_{кр1} + \dot{z}_{кр2}) - 2G_1(z_{кр1} + z_{кр2}) = 0;$$
$$(m_{кр1} + 2m_p)\ddot{z}_{кр1} + (2\beta_1 + 2\beta_p)\dot{z}_{кр1} + (2G_1 + 2G_p)z_{кр1} = P_{p1}\,;$$
$$(m_{кр2} + 2m_p)\ddot{z}_{кр2} + (2\beta_1 + 2\beta_p)\dot{z}_{кр2} + (2G_1 + 2G_p)z_{кр2} = P_{p2}\quad, \qquad (1)$$

where $P_p\,(t) = P_{p1}(t) = P_{p2}(t) = m_p\ddot{\eta}_H(t) + \beta_p\dot{\eta}_H(t) + G_p\eta_H(t)\,,\qquad (2)$

where $P_p\,(t)$ – is the dynamic load that occurs when an electric train car moves over track irregularities, and $\eta_H(t) = \eta_0 \cdot \sin\omega t$, (3)

where η_0 –is the track irregularity height, and ω is the frequency of the irregularity variation over time.

We will find the solution to system (1) using the Gauss method in the MATHCAD 15 programming environment [5,6,7].

The determinant of system (1) is

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{vmatrix} \quad (4).$$

and the solutions for z_{ak} , $\varphi_{ayк}$, $z_{aт1}$, $z_{aт2}$, z_{akp1} and z_{akp2} are, respectively,

$$\Delta1=\begin{vmatrix} b_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ b_2 & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ b_3 & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ b_4 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ b_5 & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ b_6 & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{vmatrix}, \quad \Delta2=\begin{vmatrix} a_{11} & b_1 & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & b_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & b_3 & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & b_4 & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & b_5 & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & b_6 & a_{63} & a_{64} & a_{65} & a_{66} \end{vmatrix},$$

$$\Delta3=\begin{vmatrix} a_{11} & a_{12} & b_1 & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & b_2 & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & b_3 & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & b_4 & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & b_5 & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & b_6 & a_{64} & a_{65} & a_{66} \end{vmatrix}, \quad \Delta4=\begin{vmatrix} a_{11} & a_{12} & a_{13} & b_1 & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & b_2 & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & b_3 & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & b_4 & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & b_5 & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & b_6 & a_{65} & a_{66} \end{vmatrix},$$

$$\Delta5=\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & b_1 & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & b_2 & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & b_3 & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & b_4 & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & b_5 & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & b_6 & a_{66} \end{vmatrix}, \quad \Delta6=\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & b_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & b_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & b_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & b_4 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & b_5 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & b_6 \end{vmatrix} \quad (5),$$

where

$$z_{ak}=\frac{\Delta1}{\Delta};\quad \varphi_{ayк}=\frac{\Delta2}{\Delta};\quad z_{aт1}=\frac{\Delta3}{\Delta};$$

$$z_{aт2}=\frac{\Delta4}{\Delta};\; z_{akp1}=\frac{\Delta5}{\Delta};\; ;\; z_{akp1}=\frac{\Delta6}{\Delta} \quad (6).$$

The system of differential equations (1) is solved by the Gauss matrix method using the MathCAD 15 programming environment. As a result, the amplitude-frequency response of the "car body-bogie-track" system is investigated considering the influence of spring suspension using for example the high-speed electric train AFROSIAB, moreover, the first and second bogies vibrate with different amplitudes, and the wheel pairs also vibrate differently.

Graphs are plotted for the bouncing z_k and galloping $\varphi_{yk} \approx 0$ oscillations of the car body of the electric train, and for the bouncing oscillations of wheelsets $z_т$ (Figure 2).

Based on the presented mathematical model using formulas (1)÷(6), considering the actual dimensions of the spring suspension of the high-speed electric train AFROSIAB, a numerical calculation was performed to justify rational parameters and to build the amplitude-frequency response of the "track-wheel-bogie-body" system.

Based on the theoretical and numerical studies conducted, the following general conclusions can be drawn:

We developed an algorithm and a program for the MathCad 15 programming environment to describe the vertical oscillations of the high-speed electric train AFROSIAB. We then conducted numerical studies for the proposed mathematical model.

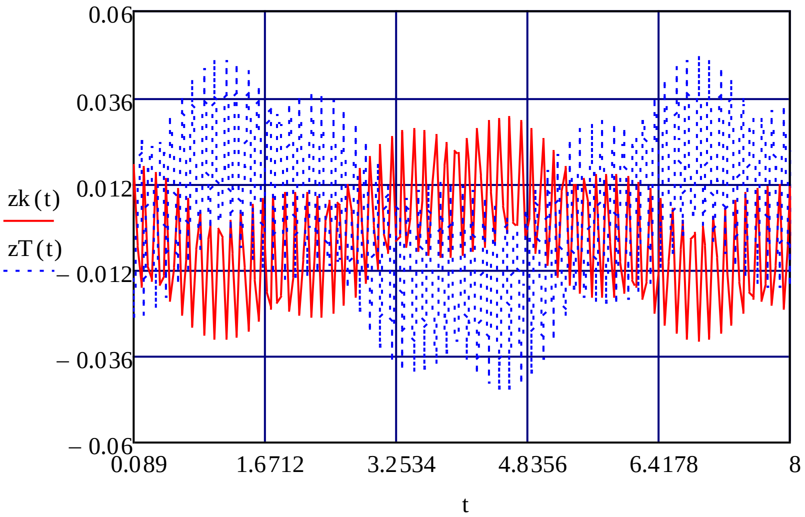

Figure 2. Graph of bouncing $z_k(t)$ oscillations of the body of the electric train car and recoiling oscillations of wheelsets $z_T(t)$.

As a result, we developed an analytical and numerical model using a method similar to the Gauss method. This model allows us to analyze the amplitude-frequency spectrum of vertical oscillations of the high-speed electric train AFROSIAB.

Based on the numerical results, we identified the most dangerous zones, where the amplitudes of vertical oscillations are most considerable. It is evident (see Figure 2) that different vertical oscillations of the wheelsets $\boldsymbol{z_{kp1}}$ and $\boldsymbol{z_{kp2}}$ cause significant bouncing oscillations of the first and second bogies $\boldsymbol{z_{T1}}$, $\boldsymbol{z_{T2}}$. These oscillations are then transferred to the car body of the high-speed electric train AFROSIAB within the "track-wheel-bogie-body" system, leading to a deterioration in its smoothness. Therefore, it is necessary to implement pneumatic spring suspension in the central stage to optimize the functions of the spring suspension and enhance its elastic-dissipative properties at high speeds of the electric rolling stock.

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