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THEORETICAL STUDIES ON THE BASIS, VECTOR COORDINATES, AND AFFINE COORDINATE SYSTEMS

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ABSTRACT

This article provides a theoretical analysis of the fundamental concepts of linear algebra and affine geometry, specifically focusing on bases, vector coordinates, and affine coordinate systems. The study outlines the conditions of linear independence and completeness of a basis in vector spaces, as well as the transformation laws of coordinates upon changing the basis. Furthermore, it examines the relationship between points and vectors in affine space, and addresses the issues of coordinate system transformation through relevant theorems and their formal proofs. The article highlights the significance of mathematical apparatus in understanding linear and affine space structures, serving to develop fundamental geometric insights.

BAZIS VA VEKTORNING KOORDINATALARI. AFFIN KOORDINATALAR SISTEMALARI HAQIDA NAZARIY TABDIQLAR

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Bazis, fazoning o’lchov "skeleti, bazis vektorlarining vazn koeffitsiyentlari, o’tish matritsasi, bir koordinatalar sistemasidan ikkinchisiga o’tish ko’prigi, Chiziqli fazo, bazis, vektor koordinatalari, affin koordinatalar sistemasi, o’tish matritsasi, radius-

ABSTRACT

Ushbu maqolada chiziqli algebra va affin geometriyasining fundamental tushunchalari - bazis, vektor koordinatalari hamda affin koordinatalar sistemasi nazariy jihatdan tahlil qilinadi. Ishda vektor fazolarida bazisning chiziqli erklilik va to’liqlik shartlari, shuningdek, bazis almashtirilganda koordinatalarning o’zgarish qonuniyatlari bayon etilgan. Shuningdek, affin fazoda nuqta va vektor tushunchalari orasidagi bog’liqlik, koordinatalar sistemasini o’zgartirish (transformatsiya) masalalari tegishli teoremalar va ularning isbotlari orqali yoritib berilgan. Maqola chiziqli fazo va affin fazo tuzilmalarini anglashda matematik apparatning ahamiyatini ko’rsatib, fundamental geometrik tasavvurlarni shakllantirishga xizmat qiladi.

vektor, chiziqli erkli Sistema.

Ushbu maqolaning maqsadi - chiziqli fazolarda bazis va vektor koordinatalari nazariyasini tizimli bayon etish, affin koordinatalar sistemasining geometrik mohiyatini yoritish hamda koordinatalarni o'zgartirish jarayonlarida qo'llaniladigan o'tish matritsalarining nazariy va amaliy asoslarini tahlil qilishdir. Maqola doirasida asosiy ta'riflar keltirilib, ular tegishli teoremalar va ularning matematik isbotlari bilan asoslab beriladi. Bu orqali o'quvchida fazoviy tuzilmalar haqidagi fundamental tasavvurlarni mustahkamlash va amaliy masalalarda ulardan samarali foydalanish ko'nikmalarini shakllantirish ko'zda tutilgan.

Ta'rif 1 (Chiziqli bog'liqlik va erklik): V vektor fazosidagi $\{e_1, e_2, \dots, e_n\}$ vektorlar sistemasi uchun quyidagi tenglikni qaraymiz:

$$\alpha_1 e_1 + \alpha_2 e_2 + \dots + \alpha_n e_n = 0$$

Agar bu tenglik faqat $\alpha_1 = \alpha_2 = \dots = \alpha_n = 0$ bo'lganda bajarilsa, bu sistema **chiziqli erkli** deyiladi. Agar hech bo'lmaganda bitta $\alpha_i \neq 0$ bo'lsa, sistema **chiziqli bog'liq** deyiladi.

Ta'rif 2 (Bazis): V fazodagi $\{e_1, \dots, e_n\}$ sistemasi V uchun bazis deyiladi, agar:

1. Sistema chiziqli erkli bo'lsa.
2. Fazodagi har qanday x vektorni $x = \sum_{i=1}^{n} \alpha_i e_i$ ko'rinishida yozish mumkin bo'lsa.

Teorema: Koordinatalarning yagonaligi. Agar $\{e_1, \dots, e_n\}$ bazis bo'lsa, har bir $x \in V$ vektorni ushbu bazis bo'yicha yagona usulda yoyish mumkin. Ya'ni, x vektor uchun $\{\alpha_1, \dots, \alpha_n\}$ koeffitsiyentlar to'plami yagonadir.

Isbot: Faraz qilaylik, x vektor ikkita yoyilmasiga ega:

$$x = \sum_{i=1}^{n} \alpha_i e_i \quad \text{va} \quad x = \sum_{i=1}^{n} \beta_i e_i$$

Bu ikki tenglikni ayiramiz:

$$\sum_{i=1}^{n} \alpha_i e_i - \sum_{i=1}^{n} \beta_i e_i = 0 \Rightarrow \sum_{i=1}^{n} (\alpha_i - \beta_i) e_i = 0$$

Bazisning ta'rifiga ko'ra, $\{e_1, \dots, e_n\}$ chiziqli erkli, demak, barcha koeffitsiyentlar nol bo'lishi shart: $\alpha_i - \beta_i = 0 \Rightarrow \alpha_i = \beta_i \quad (i = 1, 2, \dots, n)$.

Demak, yoyilma yagonadir.

Bazisni almashtirish (o'tish matrisasi) agar bizda ikkita basis bo'lsa

1. Eski basis: $E = \{e_1, \dots, e_n\}$
2. Yangi bazis: $E' = \{e'_1, \dots, e'_n\}$

Har bir yangi bazis vektorini eski bazis orqali ifodalash mumkin:

$$e'_j = \sum_{i=1}^{n} a_{ij} e_i$$

Bu yerdagi $A = (a_{ij})$ matritsa **o'tish matritsasi** deyiladi.

Teorema (Koordinatalarning o'zgarishi). Agar x vektorning eski bazisdagi koordinatalari $X=(\alpha_1,\dots,\alpha_n)^T$, yangi bazisdagi koordinatalari esa $X'=(\alpha_1',\dots,\alpha_n')^T$ bo'lsa, u holda ular orasidagi bog'lanish quyidagicha bo'ladi:

$$X = AX'$$

Isbot: $x=\sum_j \alpha_j' e_j'$ tenglikka e_j' ning qiymatini qo'yamiz:

$$x=\sum_j \alpha_j'(\sum_i a_{ij}e_i)=\sum_i(\sum_j a_{ij}\alpha_j')e_i$$

$x=\sum_i \alpha_i e_i$ bo'lgani uchun, koordinatalarni solishtirsak:

$$\alpha_i=\sum_{j=1}^{n} a_{ij}\alpha_j'$$

Bu matritsali ko'rinishda $X = AX'$ ga teng.

Ta'rif 3 (Affin koordinatalar sistemasi). n o'lchovli affin fazoda A affin koordinatalar sistemasi deganda O nuqta (kelib chiqish nuqtasi) va $\{e_1,e_2,\dots,e_n\}$ bazis vektorlar juftligi tushuniladi, ya'ni $(O;e_1,e_2,\dots,e_n)$.

Ta'rif 4 (Nuqtaning koordinatalari). Fazodagi ixtiyoriy M nuqtaning koordinatalari deb, shunday $(x_1,x_2,\dots,x_n)$ sonlar to'plamiga aytiladiki, bunda:

$$\overrightarrow{OM} = x_1e_1 + x_2e_2 + \dots + x_ne_n$$

Bu yerda $\overrightarrow{OM}$ vektor M nuqtaning **radius-vektori** deyiladi.

Teorema: Koordinatalarning yagonaligi. Affin koordinatalar sistemasida fazoning har bir M nuqtasi uchun yagona koordinatalar to'plami mavjud.

Isbot. Mavjudligi: Vektor fazoning ta'rifiga ko'ra, O nuqtadan fazodagi har qanday M nuqtaga $\overrightarrow{OM}$ vektor o'tkazish mumkin. Bazis ta'rifiga ko'ra, har qanday vektor bazis orqali yoyiladi. Demak, $\overrightarrow{OM}=\sum x_ie_i$ yoyilma mavjud.

Yagonaligi: Faraz qilaylik, $\overrightarrow{OM}=\sum x_ie_i$ va $\overrightarrow{OM}=\sum y_ie_i$ bo'lsin. U holda:

$$\sum x_ie_i=\sum y_ie_i \Rightarrow \sum (x_i-y_i)e_i=0$$

Bazis vektorlar chiziqli erkli bo'lgani uchun, barcha $x_i-y_i=0$, ya'ni $x_i=y_i$ bo'lishi shart.

Nuqtalar orasidagi vektor. Agar $A(a_1,\dots,a_n)$ va $B(b_1,\dots,b_n)$ nuqtalar berilgan bo'lsa, $\overrightarrow{AB}$ vektorni topish formulasi:

Teorema.

$$\overrightarrow{AB}=(b_1-a_1,b_2-a_2,\dots,b_n-a_n)$$

Isbot: Vektorlar yig'indisi qoidasiga ko'ra:

$$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$$

Ta'rifga ko'ra $\overrightarrow{OB}=\sum b_ie_i$ va $\overrightarrow{OA}=\sum a_ie_i$.

$$\overrightarrow{AB}=\sum b_ie_i-\sum a_ie_i=\sum (b_i-a_i)e_i$$

Demak, vektorning koordinatalari nuqtalar koordinatalarining ayirmasiga teng.

Affin almashtirish (Bazis va nuqta o'zgarishi). Agar biz koordinatalar sistemasini $(O;e_i)$ dan $(O';e_i')$ ga o'tkazsak, nuqtaning koordinatalari qanday o'zgaradi?

Faraz qilaylik:

1. O' nuqtaning eski sistemadagi koordinatalari $O'(c_1,\dots,c_n)$ bo'lsin

2. $e'_j = \sum a_{ij} e_i$ (o'tish matritsasi).

Teorema (Koordinatalarni qayta hisoblash). Yangi (x'_i) va eski (x_i) koordinatalar orasidagi bog'lanish:

$$x_i = c_i + \sum_{j=1}^{n} a_{ij} x'_j$$

Isbot: $\overrightarrow{OM} = \overrightarrow{OO'} + \overrightarrow{O'M}$ ekanligini bilamiz.

$$\overrightarrow{OM} = \sum x_i e_i$$

$$\overrightarrow{OO'} = \sum c_i e_i$$

$$\overrightarrow{O'M} = \sum x'_j e'_j = \sum x'_j \left(\sum a_{ij} e_i\right) = \sum_i \left(\sum_j a_{ij} x'_j\right) e_i$$

Tenglikning o'ng tomonini yig'ib chiqamiz:

$$\sum_i x_i e_i = \sum_i \left(c_i + \sum_j a_{ij} x'_j\right) e_i$$

Tadqiqot natijalari shuni ko'rsatadiki, bazis va koordinatalar tizimi chiziqli fazolarni o'rganishning "skeleti" hisoblanadi.

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