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SOLVING PROBLEMS USING BLOCK MATRIX CONSTRUCTION

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ABSTRACT
This article explains the concept of block matrix, solving problems using block matrix construction, and applying block matrices to problems with theoretical information and examples.

РЕШЕНИЕ ЗАДАЧ С ИСПОЛЬЗОВАНИЕМ БЛОЧНОГО МАТРИЧНОГО ПОСТРОЕНИЯ

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ABSTRACT
В этой статье объясняется концепция блочной матрицы, решение проблем с использованием построения блочной матрицы и применение блочной матрицы к задачам с теоретической информацией и примерами.

MASALALARNI BLOK MATRITSA QURISH YORDAMIDA YECHISH

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ABSTRACT

Ushbu maqolada blok matritsa haqida tushuncha, masalalarni blok matritsa qurish orqali yechish hamda masalalarda blok matritsalarni tatbiq qilish haqida nazariy ma’lumotlarlar hamda misollar tushuntirib ko‘rsatilgan.

Asosiy g‘oyani tushunish:

Bizga kichik o‘lchamli quyidagi matritsalarni qaraylik:

$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $B=\begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix}$, $C=\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$ va $D=\begin{pmatrix} d_{11} & d_{12} \end{pmatrix}$.

Matritsalarni ko‘paytirish va qo‘shish ta’rifiga ko‘ra

$$AC+BD=\begin{pmatrix} a_{11}c_{11}+a_{12}c_{21} & a_{11}c_{12}+a_{11}c_{12} \\ a_{21}c_{11}+a_{22}c_{21} & a_{21}c_{12}+a_{22}c_{22} \end{pmatrix}+\begin{pmatrix} b_{11}d_{11} & b_{11}d_{12} \\ b_{21}d_{11} & b_{21}d_{12} \end{pmatrix}=$$

$$=\begin{pmatrix} a_{11}c_{11}+a_{12}c_{21}+b_{11}d_{11} & a_{11}c_{12}+a_{11}c_{12}+b_{11}d_{12} \\ a_{21}c_{11}+a_{22}c_{21}+b_{21}d_{11} & a_{21}c_{12}+a_{22}c_{22}+b_{21}d_{12} \end{pmatrix}$$

tenglik bizda bor. Endi quyidagi blok matritsalarni (elementlari matritsa bo‘lgan) qaraylik. Demak, blok matritsaning oddiy matritsadan farqi elementlarida ekan.

$$M=\begin{pmatrix} A & B \\ * & * \end{pmatrix}=\begin{pmatrix} a_{11} & a_{12} & b_{11} \\ a_{21} & a_{22} & b_{21} \\ * & * & * \end{pmatrix} \text{ va } N=\begin{pmatrix} C & * \\ D & * \end{pmatrix}=\begin{pmatrix} c_{11} & c_{12} & * \\ c_{21} & c_{22} & * \\ d_{11} & d_{12} & * \end{pmatrix}$$

Dastlab oddiy ko‘paytirish orqali MN matritsaning elementlarini hisoblaylik. Keyin ularni yuqoridagi $AC+BD$ matritsa elementlari bilan qanday bog‘langanligini tekshirib ko‘ramiz. Quyidagi tenglikdan ko‘rishimiz qiyin emas.

$$(MN)_{11}=a_{11}c_{11}+a_{12}c_{21}+b_{11}d_{11}=(a_{11}c_{11}+a_{12}c_{21})+b_{11}d_{11}=(AC)_{11}+(BD)_{11}=(AC+BD)_{11}$$

$$(MN)_{12}=a_{11}c_{12}+a_{12}c_{22}+b_{11}d_{12}=(a_{11}c_{12}+a_{12}c_{22})+b_{11}d_{12}=(AC)_{12}+(BD)_{12}=(AC+BD)_{12}$$

$$(MN)_{21}=a_{12}c_{11}+a_{22}c_{21}+b_{21}d_{11}=(a_{12}c_{11}+a_{22}c_{21})+b_{21}d_{11}=(AC)_{21}+(BD)_{21}=(AC+BD)_{21}$$

$$(MN)_{21}=a_{21}c_{12}+a_{22}c_{22}+b_{21}d_{12}=(a_{21}c_{12}+a_{22}c_{22})+b_{21}d_{12}=(AC)_{22}+(BD)_{22}=(AC+BD)_{22}$$

Demak, ushbu:1

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$$MN=\begin{pmatrix} a_{11} & a_{12} & b_{11} \\ a_{21} & a_{22} & b_{21} \\ * & * & * \end{pmatrix}\times\begin{pmatrix} c_{11} & c_{12} & * \\ c_{21} & c_{22} & * \\ d_{11} & d_{12} & * \end{pmatrix}=\begin{pmatrix} A & B \\ * & * \end{pmatrix}\times\begin{pmatrix} C & * \\ D & * \end{pmatrix}=\begin{pmatrix} AC+BD & * \\ * & * \end{pmatrix}$$

tenglik o‘rinli bo‘lar ekan.

Ya’ni matritsa elementlarini guruhlab ichki matritsa hosil qilib, keyin asosiy matritsalarni ko‘paytirishda ichki matritsalarni son kabi qarab oddiy ko‘paytirish ta’rifidan foydalanish mumkin. Lekin bunda e’tibor berishimiz muhim bo‘lgan ikkita jihati bor.

1. Ichki matritsalarni o‘zaro ko‘paytirish mumkin bo‘lishi kerak (ichki matritsalarning birinchisining ustunlar soni ikkinchisining satrlar soniga teng bo‘lishi kerak).
2. Ichki matritsalarning ko‘paytirish tartibini saqlash kerak.

Oddiy sonlardan farqi shu yerda. Qolganini masalalar yechish davomida ko‘ramiz.

Qo‘llaniladigan asosiy tenglik va tengsizliklar:

Lemma 1. $A,B\in M_{n\times n}(C)$ lar uchun $AB=I_n$ bo‘lsa $BA=I_n$ bo‘ladi

(bizga ma’lumki bu yerda I_n birlik matritsa).

Lemma 2. $A,B\in M_{n\times n}(C)$ lar uchun, $\det\begin{pmatrix} A & * \\ 0 & B \end{pmatrix}=\det(A)\times\det(B)$ tenglik o‘rinli bo‘ladi.

Lemma 3. $A\in M_{s\times n}(C),\ B\in M_{n\times m}(C)$ bo‘lsa,

$$r\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}=r(A)+r(B)$$

tenglik o‘rinli (bu yerda $r(M)$ M matritsaning rangi).

Lemma 4. $A\in M_{s\times n}(C),\ B\in M_{n\times m}(C)$ lar uchun ushbu $AB=0$ tenglik o‘rinli bo‘lsa $r(A)+r(B)\le n$ bo‘ladi.

Isbot. Faraz qilaylik $B=(B_1,B_2,...,B_m)=0$ ko‘rinishda bo‘lsin bundan $AB=0$ shartga ko‘ra $A(B_1,B_2,...,B_m)=0$ tenglikka egamiz bu esa $AB_i=0$ $i=\overline{1,m}$ ifodaga teng kuchli. Demak B_i $\left(i=\overline{1,m}\right)$ lar bir jinsli bo‘lgan $AX=0$ tenglama yechimlari ekan. Endi $AX=0$ tenglamalar sistemasining umumiy yechimlari soni $n-r(A)$ dan oshmasligini inobatga olib, $r(B)\le n-r(A)$ tengsizlikka ega bo‘lamiz. Bu esa $r(A)+r(B)\le n$ tengsizlikka teng kuchli.

Lemma 5. $A\in M_{s\times n}(C),\ B\in M_{n\times m}(C)$ uchun quyidagi tengsizlik o‘rinli bo‘ladi:

$$r(A)+r(B)\le r\begin{pmatrix} A & * \\ 0 & B \end{pmatrix}\le\min\left(r(A)+m,\ r(B)+s\right).$$

Lemma 6. $A\in M_{s\times n}(C),\ B\in M_{n\times m}(C)$ uchun quyidagi tengsizlik o‘rinli bo‘ladi:

$$r(AB)\le\min\left(r(A),r(B)\right).$$

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Misol 1. $\exists ABC, AB, BC\left(A,B,C \in M_{n\times n}(R)\right)$ lar mavjud bo‘lsa u holda quyidagi $r(ABC)+r(B)\geq r(AB)+r(BC)$ tengsizlikni isbotlang.

Yechim. Ushbu blok matritsani tuzib olamiz:

$$M=\begin{pmatrix} B & 0 \\ 0 & ABC \end{pmatrix} \ (1).$$

Lemma 3 ga ko‘ra

$$r(M)=r(B)+r(ABC) \ (2)$$

tenglik o‘rinli.

Endi $(1)-$ blok matritsani chiziqli almashtirishlar yordamida ko‘rinishini o‘zgartiramiz. Chiziqli almashtirishni quyidagi tartibda bajaramiz:

$1-$ *qadam.* Blok matritsaning $1-$ satrini A matritsaga ko‘paytirib $2-$ satriga qo‘shamiz hamda hosil bo‘lgan ifodani $2-$ satrga yozamiz.

$2-$ *qadam.* Blok matritsaning $1-$ ustunini $-C$ matritsaga ko‘paytirib $2-$ ustunga qo‘shamiz hamda hosil bo‘lgan ifodani $2-$ ustunga yozamiz:

$$M=\begin{pmatrix} B & 0 \\ 0 & ABC \end{pmatrix}\Rightarrow\begin{pmatrix} B & 0 \\ AB & ABC \end{pmatrix}\Rightarrow\begin{pmatrix} B & -BC \\ AB & 0 \end{pmatrix}\Rightarrow-\begin{pmatrix} B & BC \\ AB & 0 \end{pmatrix}\Rightarrow\begin{pmatrix} AB & B \\ 0 & BC \end{pmatrix}.$$

Chiziqli almashtirishlar yordamida hosil bo‘lgan yangi blok matritsani 5-lemmaga ko‘ra $r\begin{pmatrix} AB & B \\ 0 & BC \end{pmatrix}\geq r(AB)+r(BC)$ ko‘rinishda yoza olamiz. Bu esa bizga quyidagi tenglikni beradi:

$$r(M)\geq r(AB)+r(BC).$$

Ushbu ifodadagi $r(M)$ ning o‘rniga (2) tenglikni qo‘yib umumlashtirsak:

$$r(B)+r(ABC)\geq r(AB)+r(BC)$$

ifodaga ega bo‘lamiz. Demak tengsizlik isbotlandi.

Misol 2. $A\in M_{s\times n}(C),\ B\in M_{n\times m}(C)$ bo‘lsa, $r(AB)+n\geq r(A)+r(B)$ (Silvester tengsizligi) tengsizlik o‘rinli ekanligini isbotlang.

Yechim. Ushbu blok matritsani tuzib olamiz:

$$M=\begin{pmatrix} I_n & 0 \\ 0 & AB \end{pmatrix} \ (3).$$

Lemma 3 ga ko‘ra:

$$r(M)=r(I_n)+r(AB) \ (4).$$

Endi $(3)-$ blok matritsani chiziqli almashtirishlar yordamida ko‘rinishini o‘zgartiramiz:

$$M=\begin{pmatrix} I_n & 0 \\ 0 & AB \end{pmatrix}\Rightarrow\begin{pmatrix} I_n & 0 \\ A & AB \end{pmatrix}\Rightarrow\begin{pmatrix} I_n & -B \\ A & 0 \end{pmatrix}\Rightarrow\begin{pmatrix} B & I_n \\ 0 & A \end{pmatrix}.$$

Chiziqli almashtirishlar yordamida hosil bo‘lgan yangi blok matritsani *Lemma 5* ga ko‘ra:

$$r(M)\geq r(A)+r(B).$$

Ushbu ifodadagi $r(M)$ ning o‘rniga (4) tenglikni qo‘yib umumlashtirsak:

$r(I_n)+r(AB)\geq r(A)+r(B)$.

Ushbu tengsizlikdagi (I_n) birlik matritsa $r(I_n)=n$ ekanligidan quyidagi ko‘rinishga kelamiz.

$n+r(AB)\geq r(A)+r(B)$

tengsizlik isbotlandi

Misol 3. $A\in M_{n\times n}(C)$ $A^2=A$ bo‘lganda ushbu:

$r(A)+r(I_n-A)=n$

tenglik o‘rinli ekanligini ko‘rsating.

Yechim. Eng avval blok matritsa tuzib olamiz:

$$M=\begin{pmatrix} A & 0 \\ 0 & I_n-A \end{pmatrix}.$$

Ushbu blok matritsamiz *Lemma 3* ga ko‘ra $r(M)=r(A)+r(I_n-A)$ ga teng.

Endi tuzgan blok matritsamizni chiziqli almashtirishlar yordamida ko‘rinishini o‘zgartiramiz:

$$M=\begin{pmatrix} A & 0 \\ 0 & I_n-A \end{pmatrix}\Rightarrow\begin{pmatrix} A & 0 \\ A & I_n-A \end{pmatrix}\Rightarrow\begin{pmatrix} A & A \\ A & I_n \end{pmatrix}\Rightarrow\begin{pmatrix} A-A^2 & 0 \\ A & I_n \end{pmatrix}\Rightarrow\begin{pmatrix} A-A^2 & 0 \\ 0 & I_n \end{pmatrix}.$$

Almashtirishlar orqali hosil bo‘lgan matritsamizni *Lemma 3* ga ko‘ra quyidagi:

$$r\begin{pmatrix} A-A^2 & 0 \\ 0 & I_n \end{pmatrix}=r(A-A^2)+r(I_n)$$

$r(M)=r(A-A^2)+r(I_n)$.

ko‘rinishga keldik.

$r(A)+r(I_n-A)=r(A-A^2)+r(I_n)$

$A^2=A$ ekanligidan:

$-A^2+A=0\Rightarrow r(-A^2+A)=0$.

Barcha tengliklarni umumlashtirib yozganimizda quyidagiga teng bo‘ladi:

$r(A)+r(I_n-A)=r(I_n)$.

Bu esa biz izlagan natija:

$r(A)+r(I_n-A)=n$.

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