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MATRIX COLOR AND ITS APPLICATIONS

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ABSTRACT

This article is devoted to the color of the matrix, and it is shown that the maximum number of linear vectors between the column vectors of the matrix is equal to the maximum number of linear vectors between the row vectors.

МАТРИЦА ЦВЕТА И ЕГО ПРИМЕНЕНИЕ

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ABSTRACT

Данная статья посвящена цвету матрицы и показано, что максимальное количество линейных векторов между векторами-столбцами матрицы равно максимальному количеству линейных векторов между векторами-строками.

MATRITSA RANGI VA UNING TADBIQLARI

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ABSTRACT

Ushbu maqola Matritsa rangiga bag'ishlangan bo'lib,matritsa ustun vektorlari orasidagi chiziqli erkli vektorlarning maksimal soni satr vektorlari orasidagi chiziqli vektorlarning maksimal soniga tengligi ko'rsatilgan.

Quyida Matritsa ustunlari fazosining rangi satrlar fazosining rangiga tengligini yana aniqroq qilib aytganda ustun vektorlari orasidagi chiziqli erkli vektorlarning maksimal soni satr vektorlari orasidagi chiziqli vektorlarning maksimal soniga tengligini ko'rsatamiz.

Ta'rif: A matritsaning ustun fazosi deb uning ustun vektorlarining chiziqli kombinatsiyalaridan tuzilgan vektor fazoga aytiladi.Ustun fazoning rangi deganda bu fazoning o'lchamini ya'ni A ning ustun vektorlari orasidagi chiziqli erkli vektorlarning maksimal sonini tushunamiz. A matritsaning satr fazosi va uning rangi ham yuqoridagidek ta'riflanadi.Yuqoridagi ta'rifda ustun so'zini ta'rifga almashtiramiz.

Misol:

$$A=\begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

Ko'rish qiyin emaski A ning ustun vektorlari orasida ham satr vektorlari orasida ham 3 tasi chiziqli erkli.Endi savol tug'iladi,ustun fazoning o'lchami har doim ham satr fazoning o'lchamiga tengmi? Javob: Ha.

Quyida buni 2 xil usulda isbotlaymiz.

1. *Chiziqli kombinatsiyalar usuli:* Bizga biror $A_{m\times n}$ matritsa berilgan bo'lsin. A ning ustun fazosining o'lchami r va bazisi $c_1, c_2, c_3, \dots, c_r$ bo'lsin. (Albatta bunda c lar A matritsaning ustun vektorlari ichidan olingan). Demak, biz A ning ixtiyoriy ustun vektorini ▲larning chiziqli kombinatsiyasi ko'rinishida ifodalay olamiz, a_i, $i=1,2,3,\dots,n$ lar A ning ustun vektorlari desak

$$a_i=\sum_{k=1}^{r} x_{k,i}c_k$$

Bundan

$$A=\left(a_1\,|\,a_2\,|\dots|\,a_n\,|\right)=\left(\sum_{k=1}^{r} x_{k,1}c_k \left|\sum_{k=1}^{r} x_{k,2}c_k\right|\dots\left|\sum_{k=1}^{r} x_{k,n}c_k\right|\right)=c_{m\times r}x_{r\times n}.$$

Bu yerda c $c_1,c_2,c_3,\ldots,c_r$ ustunlardan, x esa $x_{i,j}$ $1\le i\le r$, $1\le j\le n$ elementlardan tuzilgan matritsalar. A satr fazosining o‘lchamini S deylik. A matritsaga boshqa tomondan qarasak $A=cx$ tenglikka ko‘ra A ning har bir satr vektori x matritsaning v satr vektorlarining chiziqli kombinatsiyalaridan iborat bo‘lib qolyapdi. Bundan kelib chiqadiki A ning satr fazosi x matritsaning r ta satr vektorlaridan tuzilgan vektor fazoning qism fazosi bo‘lar ekan va qism fazoning o‘lchamidan oshmasligi haqidagi teoremani hisobga olsak $s\le r$.

Endi huddi shu mulohazalarni A matritsaning transponirlangani uchun yuritamiz va A^T ning ustun fazosi o‘lchami A ning satr fazosi o‘lchamiga, A^T ning satr fazosi o‘lchami A ning ustun fazosi o‘lchamiga tengligiga ko‘ra $r\le s$ munosabatni topamiz. Demak, $r=s$. Bundan ko‘rinadiki, ixtiyoriy matritsaning chiziqli erkli ustunlarining maksimal soni chiziqli erkli satrlarning maksimal soniga teng bo‘lar ekan. ▲

2. Satr fazoning o‘lchami r bo‘lsin. c_i, $1\le i\le r$ lar esa satr fazoning bazisi (Albatta c_i, $1\le i\le r$ larni A ning satr vektorlari deb olamiz.) Avval

$A_{c_1}, A_{c_2}, A_{c_3},\ldots,A_{c_r}$ larning ham chiziqli erkli ekanligini ko‘rsatamiz.

$c_1,c_2,c_3,\ldots,c_r$ larning chiziqli erkli ekanligidan bilamizki,

$$x_1c_1+x_2c_2+x_3c_3+\ldots+x_rc_r=0\equiv x_1=0,\ \mathrm{x}_2=0,\ \mathrm{x}_3=0,\ \ldots,\ \mathrm{x}_r=0 \qquad (*)$$

$$x_1Ac_1+x_2Ac_2+x_3Ac_3+\ldots+x_rAc_r=0$$

bo‘lsin. U holda:

$$A\left(x_1c_1+x_2c_2+x_3c_3+\ldots+x_rc_r\right)=0,$$

$v=x_1c_1+x_2c_2+x_3c_3+\ldots+x_rc_r$ desak $Av=0$.

Bu tenglikdan kelib chiqadiki v vektor A ning barcha satrlari bilan ortogonal va demak bu satrlarning ixtiyoriy chiziqli kombinatsiyasi bilan ya’ni A ning satr fazosi bilan ortogonaldir. Boshqa tomondan v tuzilishiga ko‘ra u satr fazo bazis vektorlarining chiziqli kombinatsiyasidan iborat va bundan u ushbu fazoga tegishli bo‘ladi. Demak, v vektor o‘z-o‘ziga ham ortogonal, ya’ni

$(v,v)=\left|v^2\right|=0,$ (*) ga ko‘ra $x_1=0,\ \mathrm{x}_2=0,\ \mathrm{x}_3=0,\ \ldots,\ \mathrm{x}_r=0.$ Demak, $A_{c_1}, A_{c_2}, A_{c_3},\ldots,A_{c_r}$ lar chiziqli erkli.

Ko‘rishimiz mumkinki har bir A_{c_i} vektor A ning ustun vektorlarining chiziqli kombinatsiyasidan tuzilgan. Demak ular ustun fazoga tegishli. Ustun fazoning o‘lchamini s deylik. Bir tomondan ustun fazoda

r ta A_{c_i} vektor bor, ikkinchi tomondan ustun fazodagi chiziqli erkli vektorlar soni s dan osholmaydi, bundan $r\le s.$ Yana xuddi shu mulohazani A ning transponirlangani uchun yuritib $s\le r$ ni, bulardan esa $r=s$ ni topamiz. ▲

Lemma 1. $A\in M_{s\times n}(C),\ B\in M_{n\times m}(C)$ bo‘lsa,

$$r\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}=r(A)+r(B)$$

tengsizlik o‘rinli bo‘ladi.

Lemma 2. $A \in M_{s\times n}(C),\ B \in M_{n\times m}(C)$ lar uchun ushbu $AB = 0$ tenglik o‘rinli bo‘lsa,

$$r(A) + r(B) \leq n$$

tengsizlik o‘rinli bo‘ladi.

Isbot. Faraz qilaylik $A(B_1, B_2, B_3, \ldots, B_m)$ ko‘rinishda bo‘lsin bundan $AB = 0$ shartga ko‘ra

$$A(B_1, B_2, B_3, \ldots, B_m) = 0$$

tenglikka egamiz. Bu esa $AB_i = 0\ i = \overline{1,m}$ ifodaga teng kuchli. Demak $B_i\ i = \overline{1,m}$ lar bir jinsli bo‘lgan $Ax = 0$ tenglama yechimlari ekan. Endi $Ax = 0$ tenglamalar sistemasining umumiy yechimlari soni $n - r(A)$ dan oshmasligini hisobga olib, $r(B) \leq n - r(A)$ tengsizlikka ega bo‘lamiz. Bu esa

$$r(A) + r(B) \leq n$$

tegsizlikka teng kuchli. ▲

Lemma 3. $A \in M_{s\times n}(C),\ B \in M_{n\times m}(C)$ bo‘lsa,

$$r(AB) \leq \min\left(r(A); r(B)\right)$$

tengsizlik o‘rinli bo‘ladi.

Lemma 4. $A \in M_{s\times n}(C),\ B \in M_{n\times m}(C)$ bo‘lsa,

$$r(A) + r(B) \leq r\begin{pmatrix} A & * \\ 0 & B \end{pmatrix} \leq \min\left(r(A) + m, r(B) + s\right)$$

tengsizlik o‘rinli bo‘ladi.

Sylvester tengsizligi.

$A \in M_{s\times n}(C),\ B \in M_{n\times m}(C)$ bo‘lsin. U holda quyidagi tengzlik o‘rinli:

$$r(AB) + n \geq r(A) + r(B).$$

Isbot. Chap tomon uchun *Lemma 1* ga ko‘ra

$$M = \begin{pmatrix} I_n & 0 \\ 0 & AB \end{pmatrix}$$

deb tanlasak, u holda $r(AB) + n = r(M)$ tenglik o‘rinli bo‘ladi. Endi bundan o‘ng tomondagi ifodani hosil qilish uchun quyidagicha „umumlashgan chiziqli almashtirish'' bajaramiz (birinchi satrni A ga (chap tomondan) ko‘paytirib ikkinchi ustunga qo‘shamiz):

$$M = \begin{pmatrix} I_n & 0 \\ 0 & AB \end{pmatrix} \to \begin{pmatrix} I_n & 0 \\ A & AB \end{pmatrix} \to \begin{pmatrix} I_n & -B \\ A & 0 \end{pmatrix} \to \begin{pmatrix} B & I_n \\ 0 & A \end{pmatrix}.$$

Demak *Lemma 4* ga ko‘ra quyidagi

$$r(AB) + n = r(M) = r\begin{pmatrix} B & I_n \\ 0 & A \end{pmatrix} \geq r(A) + r(B)$$

munosabatga egamiz. Bu esa isbotlanishi talab qilingan tengsizlik edi.

Misol. Agar $A, B \in M_n(R)$ matritsalar uchun $AB = A + B$ oʻrinli boʻlsa. U holda quyidagini isbotlang:

$$r(A^2) + r(B^2) \leq 2r(AB).$$

Yechim.

$$AB = A + B \Rightarrow (A - I)(B - I) = I \Rightarrow CC^{-1} = I \Rightarrow \det(A - I) \neq 0 \Rightarrow \det(B - I) \neq 0$$

$r(A - I) = n$ va $r(B - I) = n.$

Berilgan tengsizlikni *Sylvestr tengsizligidan* foydalanib isbotlaymiz.

$$r(B - I) + r(A^2) \leq r(I) + r(A^2(B - I))$$

$$r(A^2) \leq r(A^2B - A^2)$$

$$r(A^2) \leq r(A^2 + AB - A^2)$$

$$r(A^2) \leq r(AB)$$

$$r(A^2) + r(B^2) \leq 2r(AB)$$

berilgan tengsizlik isbotlandi. ▲

Frobeniya tengsizligi.

A, B, C matritsalar uchun $ABC,\ AB$ va BC lar aniqlangan boʻlsin. Isbotlang:

$$r(AB) + r(B) \geq r(AB) + r(BC).$$

Isbot. Chap tomon uchun *Lemma 1* ga koʻra

$$M = \begin{pmatrix} B & 0 \\ 0 & ABC \end{pmatrix}$$

deb tanlasak, u holda $r(B) + r(ABC) = r(M)$ tenglik oʻrinli boʻladi. Endi bundan oʻng tomondagi ifodani hosil qilish uchun quyidagicha „umumlashgan chiziqli almashtirish" bajaramiz:

$$M = \begin{pmatrix} B & 0 \\ 0 & ABC \end{pmatrix} \to \begin{pmatrix} B & 0 \\ AB & ABC \end{pmatrix} \to \begin{pmatrix} B & -BC \\ AB & 0 \end{pmatrix} \to \begin{pmatrix} BC & B \\ 0 & AB \end{pmatrix}.$$

Demak *Lemma 4* ga koʻra quyidagi

$$r(B) + r(ABC) = r(M) = r\begin{pmatrix} BC & B \\ 0 & AB \end{pmatrix} \geq r(AB) + r(BC)$$

munosabatga egamiz. Bu esa isbotlanishi talab qilingan tengsizlik edi.#

References:

1. Songqui Wang ect, Inequality of matrix[M] (the second press), Science press, Beijing (2006).
2. Tingming Wang, From Proof of inequality of matrix-rank by the block matrix construction (J, Higher mathematics (2008)).
3. Wajin Zhuang, Guaidance of matriz theory on skew field (MI, Science press, Beijing (2006)).

4. Ian S. Murphy, The rank of the sumof two rectangular matrices. Canad. Math. Bull. 13(1970), 384.
5. L. Mirsky, An Introduction to Linear Algebra, Oxford Univ. Press, 1955.