



## THE LOGARITHMIC FUNCTION, ITS DEFINITION AND THE FIELD OF VALUES

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### ABSTRACT

Logarithmic functions are an important concept in mathematics that are widely used in various fields. In this article, we will define the logarithmic function and explain its properties, including its domain of values. We will also provide examples of how logarithmic functions are used in real-life situations.

### Introduction:

The logarithmic function is a mathematical function that is widely used in many fields, including mathematics, science, engineering, and finance. The logarithmic function is defined as the inverse of the exponential function. It is used to solve exponential equations, to measure the intensity of earthquakes, and to describe the behavior of population growth and decay. It is closely related to the exponential function, and together, they form a significant part of the study of calculus. In this article, we will explain the definition of the logarithmic function and its domain of values.

### Definition:

A logarithmic function is defined as the inverse of an exponential function. That is, given a base 'b' and a real number 'x,' the logarithmic function of 'x' with respect to 'b' is denoted as log base 'b' of 'x' and is defined as follows:

$$\log_b(x) = y \text{ if and only if } b^y = x$$

The base 'b' can be any positive number, except for the value of 1, and 'x' is a positive real number.

### Domain of values:

The domain of a logarithmic function is the set of all positive real numbers. That is, if 'b' is a positive real number and 'x' is a positive real number, then the domain of the logarithmic function  $\log_b(x)$  is  $(0, \infty)$ .

### Properties:

The logarithmic function has several important properties that are useful in solving mathematical problems. Some of these properties include:

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b(x/y) = \log_b(x) - \log_b(y)$$

$$\log_b(x^y) = y \log_b(x)$$



$$\log_b(b) = 1$$

$$\log_b(1) = 0$$

Real-life applications:

Logarithmic functions have several real-life applications, some of which include:

Earthquake measurements: The Richter scale is a logarithmic scale that is used to measure the magnitude of an earthquake.

pH measurements: The pH scale is a logarithmic scale that is used to measure the acidity or basicity of a solution.

Decibel scale: The decibel scale is a logarithmic scale that is used to measure the intensity of sound.

### Methods:

The logarithmic function is defined as follows:

$$y = \log_b(x)$$

where  $x$  is a positive real number and  $b$  is a positive real number greater than 1. The base  $b$  is usually written as a subscript. For example, if  $b = 10$ , we write  $\log_{10}(x)$  as  $\log(x)$ .

The logarithmic function is the inverse of the exponential function. If  $y = b^x$ , then  $x = \log_b(y)$ . In other words, the logarithm of a number is the exponent to which the base must be raised to obtain that number.

The logarithmic function has several properties that make it useful in many applications. These properties include:

- The logarithm of a product is the sum of the logarithms of the factors:  $\log_b(xy) = \log_b(x) + \log_b(y)$ .
- The logarithm of a quotient is the difference between the logarithms of the numerator and denominator:  $\log_b(x/y) = \log_b(x) - \log_b(y)$ .
- The logarithm of a power is the product of the exponent and the logarithm of the base:  $\log_b(x^a) = a \cdot \log_b(x)$ .

### Results:

The field of values of the logarithmic function is the set of all real numbers. This means that the logarithmic function can take on any real value. However, the logarithmic function is only defined for positive real numbers. This is because the logarithmic function is the inverse of the exponential function, and the exponential function is only defined for positive real numbers.

The logarithmic function is used in many applications, including:

- Finance: The logarithmic function is used to calculate compound interest and to model the growth of investments.
- Science: The logarithmic function is used to describe the behavior of population growth and decay, radioactive decay, and the intensity of earthquakes.
- Engineering: The logarithmic function is used to measure the signal strength of electronic devices, such as cell phones and radios.
- Mathematics: The logarithmic function is used to solve exponential equations and to simplify complex expressions.

### Discussion:



The logarithmic function is a powerful tool that is used in many fields of study. It is defined as the inverse of the exponential function and is used to solve exponential equations, to measure the intensity of earthquakes, and to describe the behavior of population growth and decay. The field of values of the logarithmic function is the set of all real numbers, and it is only defined for positive real numbers. The logarithmic function has several properties that make it useful in many applications, including finance, science, engineering, and mathematics.

**In conclusion**, the logarithmic function is an essential mathematical tool that is used in many fields of study. Its properties and applications make it a valuable asset to scientists, mathematicians, engineers, and finance professionals. Understanding the logarithmic function is crucial for solving exponential equations and for analyzing exponential phenomena in the world around us. The properties of the logarithmic function are useful in solving mathematical problems, and it has several real-life applications, including earthquake measurements, pH measurements, and the decibel scale.

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