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KARRALI TRIGONOMETRIK INTEGRALLAR

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ABSTRACT

Bu maqolada karrali trigonometrik integrallar uchun bir nechta baholarni olamiz. Dastlab, eksponential ko'phad bo'lgan holda karrali trigonometrik integrallar uchun $F(x)$ *funksiyalar uchun* J *integrallarning aniq baholarini isbotlaymiz.*

Quyidagi ko'rinishdagi integral

$$J=\int_0^1 \ldots \int_0^1 \exp\{2\pi i F(x_1,\ldots,x_r)\}dx_1\ldots dx_r$$

trigonometrik integral deyiladi, bu yerda $F(x_1,\ldots,x_r)$ -r ta $x_1,\ldots,x_r$ o'zgaruvchilarning haqiqiy funksiyasi. Bunday funksiyalar analitik sonlar nazariyasida, funksiyalar nazariyasida, matematik fizikada, ehtimollar nazariyasida va matematik statistika nazariyasida uchraydi. J ga bog'liq asosiy masalalardan biri uning modulining yuqori chegarasini topish masalasidir. Buning uchun quyidagi yordamchi teorema va lemmalarni keltiramiz.

Lemma 1. Faraz qilaylik, $f(x)=\alpha_n x^n+\ldots+\alpha_1 x$ bunda $\alpha_n,\ldots,\alpha_1$ -haqiqiy sonlar bo'lib, ulardan moduli eng kattasini α orqali belgilaymiz. U holda

$$I=\int_0^1 f(x)\exp\{2\pi i f(x)\}dx$$

integral uchun

$$|I|\le \min\left(1{,}32\alpha^{-1/n}\right)$$

tengsizlik o'rinli.

Teorema 1. Faraz qilaylik, $\alpha=\max_{0\le t_1,\ldots t_r\le n}|\alpha(t_1,\ldots,t_r)|$, $\alpha(0,\ldots,0)=0$,

$$I_r=\int_0^1 \ldots \int_0^1 \exp\{2\pi i F(x_1,\ldots,x_r)\}dx_1\ldots dx_r$$

bo'lsin, bunda

$$F(x_1,\ldots,x_r)=\sum_{t_1=0}^{n}\ldots\sum_{t_r=0}^{n}\alpha(t_1,\ldots,t_r)x_1^{t_1}\ldots x_r^{t_r}.$$

U holda quyidagi baho o'rinli:

$$|I_r| \le \min\left(1, 32^r \alpha^{-1/n} \ln^{r-1}(\alpha+2)\right).$$

Isbot. $r=1$ uchun teorema o'rinli (1-lemmaga qarang). Ko'phadning o'zgaruvchilari soni bo'yicha induksiyani qo'llaymiz. Faraz qilaylik, teorema $r-1$ o'zgaruvchi uchun o'rinli bo'lsin. Uning r o'zgaruvchi uchun o'rinli ekanligini isbotlaymiz. Umumiylikka zarar yetkazmasdan absolyut qiymat bo'yicha eng katta koeffisenti noldan farqli darajali x_1 o'zgaruvchiga tegishli bo'lsin deylik. $\alpha = |\alpha(s_1,\dots,s_r)|$ bo'lsin, u holda $s_1 > 0$ bo'ladi.

$F(x_1,\dots,x_r) = \sum_{t_1=0}^{n} \dots \sum_{t_{r-1}=0}^{n} x_1^{t_1} \dots x_{r-1}^{t_{r-1}} \varphi_{t_1,\dots t_{r-1}}(x_r)$, $t = [\ln(\alpha+1)]+1$, $E_0 = \{x_r \,\|\varphi_{s_1,\dots s_{r-1}}(x_r)| \le 1\}$,
$E_k = \{x_r \,|\alpha^{(k-1)/t} < |\varphi_{s_1,\dots s_{r-1}}(x_r)| < \alpha^{k/t}\}$, $k=\overline{1,t-1}$, $E_t = \{x_r \,|\alpha^{(t-1)/t} < |\varphi_{s_1,\dots s_{r-1}}(x_r)|\}$ deylik.

E_k to'plamga tegishli bo'lgan intervallarning uzunligini $mesE_k$ orqali belgilaymiz. 1-lemmada quyidagi baho isbotlangan edi:

$$mes\{x \| f(x)| < A\} \le 4e\left(A\alpha^{-1}\right)^{1/n}.$$

Shuning uchun $mesE_k \le 4e\alpha^{(-t+k)/(tn)}$, $k = 0,1,\dots,t$ tengsizlikga egamiz.

I_r integral uchun

$$|I_r| \le mesE_0 + \sum_{k=1}^{t-1} mesE_k \max_{x_r\in E_k}\left|\int_0^1 \dots \int_0^1 \exp\{2\pi i F(x_1,\dots,x_r)\} dx_1 \dots dx_{r-1}\right| + \max_{x_r\in E_1}\left|\int_0^1 \dots \int_0^1 \exp\{2\pi i F(x_1,\dots,x_r)\}\right.$$
$$\left. dx_1 \dots dx_{r-1}\right|$$

tengsizlik o'rinli. Induksiya faraziga ko'ra

$$\max_{x_r\in E_k}\left|\int_0^1 \dots \int_0^1 \exp\{2\pi i F(x_1,\dots,x_r)\} dx_1 \dots dx_{r-1}\right| \le \min\left(1, 32^{r-1} \alpha^{-(k-1)/(tn)} \ln^{r-2}(\alpha+2)\right)$$

bo'ladi.Bundan

$$|I_r| \le 4e\alpha^{-1/n} + \sum_{k=1}^{t-1} 4e\alpha^{(-t+k)/(tn)} 32^{r-1} \alpha^{-(k-1)/(tn)} \ln^{(r-2)}(\alpha+2) + 32^{r-1}\alpha^{-(t-1)/(tn)} \ln^{r-2}(\alpha+2) \le$$
$$\le 32^r \alpha^{-1/n} \ln^{r-1}(\alpha+2)$$

ekanligi kelib chiqadi.

Bundan tashqari $|I_r| \le 1$ oddiy tengsizlikning bajarilishi ma'lum.Bu bahoni oldingisiga birlashtirib lemmaning tasdiqini hosil qilamiz.

Quyidagi lemma olingan bahoning aniqligini ko'rsatadi.

Lemma 2. Faraz qilaylik, α>1

$$I_r(\alpha) = \int_0^1 \dots \int_0^1 \exp\{2\pi i \alpha x_1^n, \dots, x_r^n\} dx_1 \dots dx_r$$

bo'lsin. U holda I_r uchun quyidan

$$|I_r(\alpha)| \geq \frac{1}{2\pi n^r (r-1)!}\alpha^{-1/n}(\ln\alpha)^{r-1}$$

baho o'rinli.

Isbot. Dastlab

$$I_r(\alpha) = \frac{(-1)^{r-1}}{(r-1)!}\int_0^1 \exp\{2\pi i\alpha x^n\}(\ln x)^{r-1}dx$$

formulaning o'rinli ekanligini ko'rsatamiz. Uni induksiya yordamida isbotlaymiz. $r=1$ uchun formula o'rinli. $r-1$ uchun to'g'ri deb faraz qilamiz.

r uchun isbot qilaylik. Farazga ko'ra

$$I_r(\alpha) = \int_0^1 I_r(\alpha x^n)dx = \int_0^1 \frac{(-1)^{r-2}}{(r-2)!}\left(\int_0^1 \exp\{2\pi i\alpha x^n y^n\}(\ln y)^{r-2}dy\right)dx$$

bo'ladi. $z = xy$ almashtirishdan so'ng

$$I_r(\alpha) = \frac{(-1)^{r-2}}{(r-2)!}\int_0^1 \frac{dx}{x}\int_0^x \exp\{2\pi i\alpha z^n\}(\ln z - \ln x)^{r-2}dz$$

ni hosil qilamiz. Oxirgi integralni bo'laklab integrallaylik:

$$I_r(\alpha) = \frac{(-1)^{r-2}}{(r-2)!}\int_0^1 (d\ln x)\int_0^x \exp\{2\pi i\alpha z^n\}\left(\sum_{k=0}^{r-2}(-1)^k\binom{r-2}{k}(\ln x)^k(\ln z)^{r-2-k}\right)dz = \frac{(-1)^{r-2}}{(r-2)!}\times$$

$$\times\sum_{k=0}^{r-2}\frac{(-1)^k}{k+1}\binom{r-2}{k}\int_0^1 (d\ln x)^{k+1}\int_0^x \exp\{2\pi i\alpha z^n\}(\ln z)^{r-2-k}dz = \frac{(-1)^{r-1}}{(r-2)!}\sum_{k=0}^{r-2}\frac{(-1)^k}{k+1}\binom{r-2}{k}\int_0^1 \exp\{2\pi i\alpha z^n\}\times$$

$$\times(\ln z)^{r-1}dz$$

$$\frac{1}{r-1}\sum_{k=0}^{r-2}\frac{(-1)^k}{k+1}(r-1)\binom{r-2}{k} = \frac{1}{r-1}$$

ekanligidan, buning r o'zgaruvchi uchun ham to'g'riligi kelib chiqadi. α>1 da $|I_r(\alpha)|$ ni quyidan baholaymiz. $J = \mathrm{Im}(I_r(\alpha))$ bo'lsin. U holda

$$J = \frac{1}{(r-1)!}\int_0^1 \sin(2\pi\alpha y^n)\left(\ln\frac{1}{y}\right)^{r-1}dy = \frac{1}{n^r(r-1)!}\int_0^1 \sin(2\pi\alpha z)\left(\ln\frac{1}{z}\right)^{r-1}z^{-1+1/n}dz =$$

$$= \frac{1}{\pi\alpha n^r(r-1)!}\int_0^1 \left(\ln\frac{1}{z}\right)^{r-1}z^{-1+1/n}d\left(\sin^2(\pi\alpha z)\right) = -\frac{1}{\pi\alpha n^r(r-1)!}\int_0^1 \sin^2(\pi\alpha z)d\left(\left(\ln\frac{1}{z}\right)^{r-1}z^{-1+1/n}\right)$$

ga ega bo'lamiz. Undan tashqari

$$J = -\frac{1}{\pi\alpha n^r(r-1)!}\int_{1/(2\alpha)}^{1+1/(2\alpha)} \cos^2(\pi\alpha z)d\left(\left(\ln\frac{1}{z-1/(2\alpha)}\right)^{r-1}(z-1/(2\alpha))^{-1+1/n}\right).$$

$$-\frac{d}{dz}\left(\left(\ln\frac{1}{z}\right)^{r-1}z^{-1+1/n}\right) \geq 0 \quad (0 < z < 1)$$

monoton kamayuvchi funksiya ekanligidan, J uchun olingan ifodalarni yig'sak.

$$2J > -\frac{1}{\pi\alpha n^r (r-1)!}\int_{1/\alpha}^{1} d\left(\left(\ln\frac{1}{z}\right)^{r-1} z^{-1+1/n}\right) = \frac{1}{\pi n^r (r-1)!}\alpha^{-1/n}(\ln\alpha)^{r-1}$$

ni hosil qilamiz. Shunday qilib,

$$|I_r(\alpha)| \geq J \geq \frac{1}{2\pi n^r (r-1)!}\alpha^{-1/n}(\ln\alpha)^{r-1}.$$

Lemma isbot bo'ldi.

Teorema 2. Faraz qilaylik, $\alpha = \max_{t_1,\dots,t_r}|\alpha(t_1,\dots,t_r)|$, $\alpha(0,\dots,0)=0$,

$$I_r = \int_0^1 \dots \int_0^1 \exp\{2\pi i F(x_1,\dots,x_r)\} dx_1 \dots dx_r$$

bo'lsin,

bunda $F(x_1,\dots,x_r) = \sum_{t_1=0}^{n_1} \dots \sum_{t_r=0}^{n_2} \alpha(t_1,\dots,t_r) x_1^{t_1}\dots x_r^{t_r}$. U holda I_r integral uchun quyidagi baho o'rinli

$$|I_r| \leq \min\left(1, 32^r \alpha^{-1/n} \ln^{r-1}(\alpha+2)\right)$$

bunda $n = \max(n_1,\dots,n_r)$.

Isbot. $F(x_1,\dots,x_r)$ ko'phadni quyidagi ko'rinishda yozish mumkin.

$F(x_1,\dots,x_r) = \sum_{t_1=0}^{n} \dots \sum_{t_r=0}^{n} \beta(t_1,\dots,t_r) x_1^{t_1}\dots x_r^{t_r}$, bu yerda $\beta(t_1,\dots,t_r)$ koeffisientlar

$$\beta(t_1,\dots,t_r) = \begin{cases} \alpha(t_1,\dots,t_r) \quad agar \quad 0 \leq t_1 \leq n_1,\dots,0 \leq t_r \leq n_r \quad bo'lsa \\ 0 \quad aks \quad holda \end{cases}$$

tengliklar bilan aniqlanadi. Faraz qilaylik, $\beta = \max_{0\leq t_1,\dots t_r \leq n}|\beta(t_1,\dots,t_r)|$ bo'lsin. U holda osongina ko'rish mumkinki, $\beta = \alpha$ tenglik o'rinli bo'ladi. Endi I_r ni 1.3-teoremaga ko'ra baholaymiz. Quyidagi

$$|I_r| \leq \min\left(1, 32^r \beta^{-1/n} \ln^{r-1}(\beta+2)\right) = \min\left(1, 32^r \alpha^{-1/n} \ln^{r-1}(\alpha+2)\right)$$

ga ega bo'lamiz. Shuni isbotlash talab qilingan edi.

Teorema 3. Faraz qilaylik, $k_1,\dots,k_r$-natural sonlar uchun $k = k_1+\dots+k_r$, $\nu = \frac{1}{k}$, $\alpha_1,\dots,\alpha_r$-haqiqiy sonlar va barcha $(x_1,\dots,x_r)$ $(0 \leq x_1,\dots,x_r \leq 1)$ nuqtalar uchun $\left|\frac{\partial^k f(x_1,\dots,x_r)}{\partial x_1^{k_1}\dots\partial x_r^{k_r}}\right| > H$ tengsizlik o'rinli bo'lsin.

Faraz qilaylik, $(\alpha_1,\dots,\alpha_r)$ yo'nalish uchun shunday $v = \binom{k+r-1}{r-1}$ topilsinki, u

a) $0 \le x_1,\ldots,x_r \le 1$ kubga qarashli va ixtiyoriy oraliqdan olingan v ning $(\alpha_1,\ldots,\alpha_r)$ yo'nalishli $\left.\frac{\partial^k f(x_1+\alpha_1 t,\ldots,x_2+\alpha_2)}{\partial t^k}\right|_{t=0}$ hosilaning monotonlik oraliqlari soni m dan oshmasin.

b) $M = \left(\frac{k!}{s_1!\ldots s_r!} \alpha_1^{s_1} \ldots \alpha_r^{s_r} \right)$, (bunda, $0 \le s_1,\ldots,s_r \le k$, $k = s_1+\ldots+s_r$ va $(\alpha_1,\ldots,\alpha_r)$ vektorlar ko'rsatilgan v vektorlar bo'yicha o'zgaradi) matrisa determinantining moduli $R>0$ dan oshmasin, va M matrisa elementlari har birining algebraik to'ldiruvchisi modul bo'yicha $T>0$ dan oshmasin. U holda

$$J = \int_0^1 \ldots \int_0^1 \exp\{2\pi i f(x_1,\ldots,x_r)\} dx_1 \ldots dx_r$$

integral uchun

$$|J| \le 6kv^{2+v} m T^v R^{-v} H^{-v}$$

baho o'rinli.

Isbot. Har bir $(\alpha_1,\ldots,\alpha_r)$ yo'nalish uchun

$$\left.\frac{\partial^k f(x_1+\alpha_1 t,\ldots,x_2+\alpha_2)}{\partial t^k}\right|_{t=0} = \sum_{\substack{s_1=0 \\ s_1+\ldots+s_r=k}}^{k} \ldots \sum_{s_r=0}^{k} \frac{k!}{s_1!\ldots s_r!} \alpha_1^{s_1} \ldots \alpha_r^{s_r} \frac{\partial^k f(x_1,\ldots,x_r)}{\partial x_1^{s_1} \ldots \partial x_r^{s_r}}$$

tenglikka egamiz. Ularni $\frac{\partial^k f(x_1,\ldots,x_r)}{\partial x_1^{s_1} \ldots \partial x_r^{s_r}}$ noma'lumlar bo'yicha chiziqli tenglamalar sistemasi sifatida qarab, $\frac{\partial^k f(x_1,\ldots,x_r)}{\partial x_1^{k_1} \ldots \partial x_r^{k_r}} = \sum_{(\alpha_1,\ldots\alpha_r)} \ldots \sum c(\alpha_1,\ldots\alpha_r) \left.\frac{\partial^k f(x_1+\alpha_1 t,\ldots,x_2+\alpha_2)}{\partial t^k}\right|_{t=0}$

larni topamiz, bunda $\sum_{(\alpha_1,\ldots\alpha_r)} \ldots \sum$ -teorema shartlaridan aniqlangan barcha $(\alpha_1,\ldots,\alpha_r)$ yo'nalishlar bo'yicha yig'indini bildiradi va $|c(\alpha_1,\ldots\alpha_r)|$ koeffisientlar TR^{-1} dan oshmaydi. Har bir $(x_1,\ldots,x_r)$ $(0 \le x_1,\ldots,x_r \le 1)$ nuqta uchun $\left| \frac{\partial^k f(x_1,\ldots,x_r)}{\partial x_1^{k_1} \ldots \partial x_r^{k_r}} \right| > H$ tengsizlik bajarilgani sababli ixtiyoriy $(x_1,\ldots,x_r)$ nuqta uchun shunday $(\alpha_1,\ldots,\alpha_r)$ yo'nalish topiladiki, unda $\left| \left.\frac{\partial^k f(x_1+\alpha_1 t,\ldots,x_2+\alpha_2)}{\partial t^k}\right|_{t=0} \right| > v^{-1} T^{-1} RH$ tensizlik bajariladi. $(\alpha_1,\ldots,\alpha_r)$ yo'nalishni biror usul yordamida tartiblaymiz va $\Omega = \{(x_1,\ldots,x_r) | 0 \le x_1,\ldots,x_r \le 1\}$ kubni o'zaro kesishmaydigan $\Omega_s (s=1,\ldots,v)$ sohalarga ajratamiz.

Birinchi Ω_1 sohaga birinchi yo'nalish bo'yicha k-tartibli hosilasi $v^{-1}T^{-1}RH$ dan katta bo'lgan barcha $(x_1,\ldots,x_r)$ nuqtalarni joylashtiramiz;

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Ikkinchi Ω_2 sohaga moduli ikkinchi yo'nalish bo'yicha *k*-tartibli hosilasi shu qiymatdan katta va 1-sohaga kirmaydigan nuqtalarni joylashtiramiz;

Uchinchisiga uchinchi yo'nalish bo'yicha *k*-tartibli hosilasining moduli $v^{-1}T^{-1}RH$ dan katta va 1- hamda 2- sohaga kirmaydigan barcha nuqtalarni joylashtiramiz va hokazo.(ayrim Ω_s sohalar bo'sh bo'lishi ham mumkin).

Har bir oraliqda Ω_s to'plamga tegishli *s*-yo'nalishga parallel *sm* dan ko'p bo'lmagan oraliqlar topilishi mumkin. Haqiqatan ham, har bir oraliq yo'nalishi $(\alpha_1,\dots,\alpha_r)$ bo'lgan ixtiyoriy *v* ga parallel, Ω_1 ga tegishli va teorema shartlarini qanoatlantiruvchi oraliqlar soni *m* dan oshmaydi. Yasashga ko'ra Ω_2 to'plam har bir oraliqda ixtiyoriy *v* yo'nalihga parallel, ikkinchisidan boshlab, Ω_2 ga tegishli 2*m* dan oshmaydigan oraliqlar mavjud va hokazo. J integralni quyidagicha yozaylik $J = J_1 + \dots + J_v$ bunda

$$J_s = \int_{\Omega_s}\dots\int \exp\{2\pi i f(x_1,\dots,x_r)\}dx_1\dots dx_r.$$

J_s integralni baholaymiz. Chiziqli ortogonal almashtirish kiritaylik, ya'ni, y_1 o'qni s inchi $(\alpha_1,\dots,\alpha_r)$ vektorga parallel yo'naltiramiz, qolgan $y_2,\dots,y_r$ koordinata o'qlarini shunday tanlaymizki, $y_1, y_2,\dots,y_r$ koordinatalar sistemasi ortogonal va $x_1,x_2,\dots,x_r$ koordinatalar sistemasi bilan bir xil orientirlangan bo'lsin.

Ω_s soha bu almashtirishdan so'ng Ω'_s ga o'tadi va $f(x_1,\dots,x_r) = f_1(y_1,\dots,y_r)$ o'rinli bo'ladi. Har bir tayinlangan $(y_2,\dots,y_r)$ nuqtani shunday y_1 lar uchun $T(s;y_2,\dots,y_r)$ bilan ifodalaymizki, bunda $(y_1,\dots,y_r)$ nuqta Ω'_s ga tegishli bo'lsin. $T(s;y_2,\dots,y_r)$ to'plam sm dan ko'p bo'lmagan oraliqlardan tashkil topgan. Ω'_s ga tegishli bo'lgan $(y_1,\dots,y_r)$ nuqtaning $y_2,\dots,y_r$ o'zgaruvchilarining o'zgarish sohasini ω_s orqali belgilaymiz.

$$|J_s| = \left|\int_{\omega_s}\dots\int \int_{T(s;y_2,\dots,y_r)} \exp\{2\pi i f_1(y_1,\dots,y_r)\}dy_1\dots dy_r\right| \le \int_{\omega_s}\dots\int \left|\int_{T(s;y_2,\dots,y_r)} \exp\{2\pi i f_1(y_1,\dots,y_r)\}dy_1\right| dy_2\dots dy_r \le$$

$$\le \left|\int_{T\left(s;y_2^{(0)},\dots,y_r^{(0)}\right)} \exp\left\{2\pi i f_1\left(y_1, y_2^{(0)}\dots, y_r^{(0)}\right)\right\}dy_1\right| \int_{\omega_s}\dots\int dy_2\dots dy_r$$

bunda $\left(y_2^{(0)},\dots,y_r^{(0)}\right)$ nuqta $\left|\int_{T(s;y_2,\dots,y_r)} \exp\{2\pi i f_1(y_1,\dots,y_r)\}dy_1\right|$ funksiyaning maksimum nuqtasi. $f_1(y_1,\dots,y_r)$ dan y_1 bo'yicha olingan *k*-tartibli hosila $v^{-1}T^{-1}RH$ dan katta ekanligidan va bir o'zgaruvchili funksiyadan *k*-tartibli olingan hosilaning bahosidan (1.2-lemmadan) quyidagini hosil qilamiz:

$$|J_s| \le 6skv^v mT^v R^{-v} H^{-v}.$$

$|J_s|$ uchun baholarni yig'sak,

$$|J|=|J_1|+...+|J_v|\le\sum_{s=1}^{v}6skv^{v}mT^{v}R^{-v}H^{-v}\le 6kv^{v+2}mT^{v}R^{-v}H^{-v}$$

ni hosil qilamiz. Teorema isbot bo'ldi.

Natija.1. Faraz qilaylik, biror l yo'nalish bo'yicha $\left|\frac{\partial^k F(x_1,...,x_r)}{\partial l^k}\right|>A>0$ tengsizlik o'rinli bo'lsin va $F(x_1,...,x_r)$ funksiya 1.5-teoremaning shartlarini qanoatlantirsin, hamda $G(x_1,...,x_r)$ $0\le x_1,...,x_r\le 1$ kubda yotuvchi va l ga parallel oraliqda bo'lakli monoton va bo'lakli uzluksiz bo'lib, $|G(x_1,...,x_r)|\le H$ shartni qanoatlantirsin. U holda

$$J=\int_0^1...\int_0^1 G(x_1,...,x_r)\exp\{2\pi iF(x_1,...,x_r)\}dx_1...dx_r$$

integral uchun

$$|J|<<HA^{-1/k}$$

baho o'rinli.

Isbot. O'zgaruvchilarda chiziqli ortogonal almashtirish qilaylik (y_1 o'qni l ga parallel qilib yo'naltiraylik, qolganlarini esa birinchi koordinata sistemasi bilan yo'nalishdosh qilib yasaymiz). $0\le x_1,...,x_r\le 1$ birlik kub Ω sohaga o'tadi, va $G(x_1,...,x_r)=G_1(y_1,...,y_r)$, $F(x_1,...,x_r)=F_1(y_1,...,y_r)$.

$$J=\int_{\Omega}...\int G_1(y_1,...,y_r)\exp\{2\pi iF_1(y_1,...,y_r)\}dy_1...dy_r$$

ga egamiz.

Tayinlangan $y_2,...,y_r$ larni Ω ga tegishli bo'lgan $(y_1,...,y_r)$ nuqtalar to'plami $T(y_2,...,y_r)$ orqali belgilaymiz. $y_2,...,y_r$ larning o'zgarish sohasini ω bilan belgilaymiz. y_1 bo'yicha bo'laklab integrallasak;

$$J=\int_{\omega}...\int\left(\int_{T(y_2,...,y_r)}G_1(y_1,...,y_r)\exp\{2\pi iF_1(y_1,...,y_r)\}dy_1\right)dy_2...dy_r=$$

$$=\int_{\omega}...\int\left(\int_{T(y_2,...,y_r)}G_1(y_1,...,y_r)\frac{\partial}{\partial y_1}\left(\int_0^{y_1}\exp\{2\pi iF_1(\xi_1,y_2,...,y_r)\}d\xi_1\right)dy_1\right)dy_2...dy_r$$

$$=\int_{\omega}...\int\left(\int_0^1\exp\{2\pi iF_1(\xi_1,y_2,...,y_r)\}d\xi_1\right)G_1(1,y_2,...,y_r)dy_2...dy_r-$$

$$-\int_{\omega}...\int\left(\int_{T(y_2,...,y_r)}\left(\int_0^{y_1}\exp\{2\pi iF_1(\xi_1,y_2,...,y_r)\}d\xi_1\right)G_1(y_1,...,y_r)dy_1\right)dy_2...dy_r$$

ni hosil qilamiz. 3-teoremaga ko'ra integralni baholasak

$$\left|\int_0^{y_1}\exp\{2\pi iF_1(\xi_1,y_2,...,y_r)\}d\xi_1\right|<<A^{-1/k}$$

natijada talab qilingan baho kelib chiqadi. Natija isbot bo'ldi.

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