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PARAMETRGA BOG‘LIQ INTEGRALLAR YORDAMIDA AYRIM MUHIM INTEGRALLARNI HISOBLASH USULLARI

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KALIT SO’ZLAR

Dirixle integrali, Eyler-Puasson integrali, Laplas integrallari, Frenelli integrallari

ANNOTATSIYA

Ushbu maqolada juda ko‘p sohalarda muhim hisoblangan ayrim xosmas integrallar: Dirixle integrali, Eyler-Puasson integrali, Laplas integrallari, Frenelli integrallari yaqinlashishga tekshirilgan va ularning qiymatlarini hisoblash usullari o‘rganilgan. Ularning qiymatlarini hisoblashda parametrga bog‘liq integrallarning xossalaridan foydalanilgan.

1. Dirixle integrali.

Ushbu $J=\int\limits_0^{\infty}\frac{\sin x}{x}dx$ Dirixle integralini hisoblashning juda ko‘p usullari mavjud. Biz ushbu Dirixle integralining qiymatini parametrga bog‘liq integrallar yordamida hisoblaymiz. Buning uchun

$$J_{\alpha}=\int\limits_0^{\infty}\frac{\sin\alpha x}{x}dx,\ \left(\alpha>0\right)$$

integralni qaraymiz.

Bu intagralni α parametr bo‘yicha differensiallab hisoblaymiz. Ammo, α bo‘yicha differensiallasak,

$$\frac{dJ_{\alpha}}{d\alpha}=\int\limits_0^{\infty}\cos\alpha x\,dx$$

integral Leybnits qoidasiga ko‘ra uzoqlashuvchi bo‘ladi. Shu sababli, $e^{-kx},\ k>0$ “yaqinlashtiruvchi ko‘paytuvchi” tushunchasini kiritamiz. Buning uchun ushbu

$$J_k\left(\alpha\right)=\int\limits_0^{\infty}e^{-kx}\frac{\sin\alpha x}{x}dx,\ \left(\alpha\geq0,\ k>0\right)$$

funksiyani

$J_k\left(\alpha\right)$ funksiyadan α bo‘yicha xususiy hosila olib, quyidagini topamiz:

$$\frac{\partial J_k\left(\alpha\right)}{\partial\alpha}=\int\limits_0^{\infty}e^{-kx}\cos\alpha x\,dx=I\ .$$

Yuqoridagi integralda integral ostidagi funksiya $\alpha\in\left[0,\infty\right)$ da har doim mavjud va integral yaqinlashuvchi bo‘ladi. Uni ikki marta bo‘laklab integrallash orqali

$$I=\frac{k}{\alpha^2+k^2}$$

ekanligini topamiz.

Shunday qilib,

$$\frac{\partial J_k(\alpha)}{\partial \alpha}=\frac{k}{\alpha^2+k^2}.$$

Bundan,

$$J_k(\alpha)=arctg\frac{\alpha}{k}+C$$

bo‘lishi kelib chiqadi.

$J_k(0)=0$ munosabatdan $C=0$ va

$J_k(\alpha)=arctg\dfrac{\alpha}{k}$ ekanligi kelib chiqadi.

Agar $\alpha=const$ bo‘lsa, unda $J_k(\alpha)$ ifoda k ning funksiyasi bo‘lib qoladi. Agar $k\to+0$ da limitga o‘tsak, u holda

$$J_\alpha=\lim_{k\to+0}J_k(\alpha)=\lim_{k\to+0}\left(arctg\frac{\alpha}{k}\right)=\frac{\pi}{2}$$

tenglikka ega bo‘lamiz.

Xususan, $\alpha=1$ bo‘lganda

$$J_1=J=\int_0^\infty\frac{\sin x}{x}dx=\frac{\pi}{2}$$

ekanligi kelib chiqadi.

2. Eyler-Puasson integrali.

Bu bandda ushbu $J=\int_0^\infty e^{-x^2}dx$ – Eyler-Puasson integralining qiymatini hisoblash bilan shug‘ullanamiz. Buning uchun, dastlab $x=nt$ almashtirish bajaramiz, bu yerda n ixtiyoriy musbat son. U holda

$$J=n\cdot\int_0^\infty e^{-n^2t^2}dt$$

tengik o‘rinli bo‘ladi. Tenglikning har ikkala tomoniga e^{-n^2} ifodani ko‘paytirib, so‘ngra 0 dan ∞ gacha n bo‘yicha integrallab, quyidagi

$$J\cdot\int_0^\infty e^{-n^2}dn=\int_0^\infty e^{-n^2}n\,dn\int_0^\infty e^{-n^2t^2}dt$$

yoki

$$J=\int_0^\infty ne^{-n^2}dn\int_0^\infty e^{n^2t^2}dt$$

tenglikni hosil qilamiz. Oxirgi integralni hisoblashda o‘zgaruvchilarni o‘rinlarini almashtirib, quyidagini hosil qilamiz:

$$J^2=\int_0^\infty dt\left(\int_0^\infty e^{-(1+t^2)n^2}ndn\right)=\int_0^\infty dt\left(\frac{1}{2}\int_0^\infty e^{-(1+t^2)n^2}d(1+n^2)\right)=\frac{1}{2}\int_0^\infty dt\left(-\frac{1}{1+t^2}e^{-(1+t^2)n}\Big|_0^\infty\right)=$$

$$=\frac{1}{2}\int_0^\infty\frac{1}{1+t^2}dt=\frac{1}{2}arctgt\Big|_0^\infty=\frac{\pi}{4}$$

Bundan

$$J=\frac{\sqrt{\pi}}{2}$$

tenglik o‘rinli ekanligi kelib chiqadi.

3. Laplas integrallari.

Ushbu

$$L_1=\int_0^\infty\frac{\cos\beta x}{\alpha^2+x^2}dx \quad \text{va}$$

$$L_2=\int_0^\infty\frac{x\sin\beta x}{\alpha^2+x^2}dx,\quad(\alpha,\ \beta>0)$$

integrallar Laplas integrallari deb ataladi.

Ushbu

$$\int_0^\infty e^{-t\left(\alpha^2+x^2\right)}dt=\frac{1}{\alpha^2+x^2}$$

tenglikdan foydalanib,

$$L_1=\int_0^\infty\cos\beta x dx\int_0^\infty e^{-t\left(\alpha^2+x^2\right)}dt$$

tenglikni hosil qilamiz. Integrallash tartibini o‘zgartirib, quyidagi

$$L_1=\int_0^\infty e^{-\alpha^2t}dt\left(\int_0^\infty e^{-tx^2}\cos\beta x\,dx\right)=\int_0^\infty J(t)\cdot e^{-\alpha^2t}dt$$

tenglikni hosil qilamiz. Bu yerda

$$J(t)=\int_0^\infty e^{-tx^2}\cos\beta x\,dx$$

integralni hisoblash usuli [2] da ko‘rsatilganiga ko‘ra,

$$J(t)=\frac{1}{2}\sqrt{\frac{\pi}{t}}\cdot e^{-\frac{\beta^2}{4t}}$$

munosabatni topamiz. U holda [2] ga ko‘ra,

$$L_1=\frac{\sqrt{\pi}}{2}\int_0^\infty e^{-\alpha^2 t-\frac{\beta^2}{4t}}\cdot\frac{dt}{\sqrt{t}}$$

bo‘ladi.

Agar $t=z^2$ almashtirishni kiritsak,

$$L_1=\sqrt{\pi}\cdot\int_0^\infty e^{-\alpha^2 z^2-\frac{\beta^2}{4z^2}}dz=e^{-2\alpha\beta}\int_0^\infty e^{-\left(\alpha z-\frac{\beta}{z}\right)^2}dz=\frac{\sqrt{\pi}}{\alpha}e^{-\alpha\beta}\cdot\int_0^\infty e^{-y^2}dy=\frac{\pi}{2\alpha}\cdot e^{-\alpha\beta}$$

munosabatni hosil qilamiz.

Shunday qilib,

$$L_1=\frac{\pi}{2\alpha}\cdot e^{-\alpha\beta}$$

tenglik o‘rinli bo‘lar ekan.

Agar

$$L_2=-\frac{dL_1}{d\beta}$$

munosabatni inobatga olsak, L_2 uchun ushbu tenglikka ega bo‘lamiz:

$$L_2=\frac{\pi}{2}e^{-\alpha\beta}\ .$$

4. Frenelli integrallari.

Ushbu

$$F_1=\int_0^\infty \sin x^2 dx \text{ va } F_2=\int_0^\infty \cos x^2 dx$$

integrallar Frenelli integrallari deb ataladi. Quyidagi

$$\left|\sin x^2\right|\le 1,\ \ \left|\cos x^2\right|\le 1$$

tengsizliklar o‘rinli bo‘lganligi sababli F_1 va F_2 integrallar yaqinlashuvchi integrallardir. Ularning qiymatini hisoblash uchun $x^2=t$ almashtirish bajaramiz. Hosil bo‘lgan

$$F_1=\int_0^\infty \sin x^2 dx=\frac{1}{2}\int_0^\infty\frac{\sin t}{\sqrt{t}}dt$$ va

$$F_2=\int_0^\infty \cos x^2 dx=\frac{1}{2}\int_0^\infty\frac{\cos t}{\sqrt{t}}dt$$

integrallar ostidagi $\frac{1}{\sqrt{t}}$ ifodani quyidagi integral bilan almashtiramiz:

$$\frac{1}{\sqrt{t}}=\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-tn^2}dn\ .$$

Natijada quyidagi tenglik hosil bo‘ladi:

$$\int_0^\infty\frac{\sin t}{\sqrt{t}}dt=\frac{2}{\sqrt{\pi}}\int_0^\infty \sin t\,dt\int_0^\infty e^{-tn^2}dn\ .$$

Oxirgi integralda integrallarning o‘zgaruvchilarini almashtiramiz:

$$F_1=\int_0^\infty\frac{\sin t}{\sqrt{t}}dt=\frac{2}{\sqrt{\pi}}\int_0^\infty \sin t\,dt\int_0^\infty e^{-tn^2}dn=\frac{2}{\sqrt{\pi}}\int_0^\infty dn\left(\int_0^\infty e^{-tn^2}\sin t\,dt\right)=\frac{2}{\sqrt{\pi}}\int_0^\infty\frac{dn}{1+n^2}=\frac{2}{\sqrt{\pi}}\cdot\frac{\pi}{2\sqrt{2}}=\sqrt{\frac{\pi}{2}}$$

tenglikni hosil qilamiz.

Xuddi shuningdek, ushbu tenglikni hosil qilamiz $F_2=\sqrt{\frac{\pi}{2}}$.

FOYDALANILGAN ADABIYOTLAR:

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