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ABOUT EXPLORING THE SPECIFIC VALUES OF THE FRIDRIXS MODEL WITH MATHEMATICAL PACKAGES

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ABSTRACT

In some important problems of mathematical physics, hydrodynamics, solid state physics, quantum field theory, statistical physics and nonrealistic quantum mechanics, it is important to study the spectral properties of the Friedrich operator in several dynamic problems. in statistical physics, the lattice gas represents the binding state, and in quantum mechanics, the specific values represent the binding states of the energies. Furthermore, the three- and multi-particle systems that emerge in nonreiltivistic quantum mechanics are inextricably linked with the spectral properties of Hamiltonians.

FRIDRIXS MODELI XOS QIYMATLARINI MATEMATIK PAKETLAR YORDAMIDA TADQIQ QILISH HAQIDA

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KALIT SO'ZLAR

zarrachalar sistemasi, Fridrixs modeli, xos qiymat, Fredgolm determinanti, matematik paketlar, Maple, MathCad.

ANNOTATSIYA

Matematik fizikaning ba'zi muxim masalalarida, gidrodinamika masalalarida kattik jismlar fizikasi, kvant maydoni nazariyasi, statistik fizika va norealyavistlik kvant mexanikasida bir kancha dinamik masalalarda Fridrixs operatori spektral xossalarini urganish muxim axamiyat kasb etadi, Fridrixs operatori xos kiymatiga mos bulgan xos funksiya gidrodinamikada suyuqlik xarakatini, statistik fizikada panjaraviy gaz boglanganlik xolatini, kvant mexanikasida xos kiymatlar boglanganlik xolatlarini energiyalarini ifodalaydi. Bundan tashkari noreyltivistik kvant mexanikasida paydo buluvchi uch va kup zarrachali sistema xamiltonianlarini spektral xossalari bilan uzviy boglangandir.

$L_2([a,b]^n)$ fazoda aniqlangan quyidagi umumlashgan Fridrixs modeli H operatorni qaraymiz

$$H_\mu = H_0 - \mu V, \ \ \mu > 0 \tag{1}$$

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H_0 operator $u(x_1,,\cdots,x_n)$ funksiyaga ko‘paytirish operatori:

$$(H_0 f)(x) = (1-\cos x) f(x), \quad f \in L_2[-\pi,\pi], \qquad (2)$$

bunda $\varphi_1(x_1,\cdots,x_n), \ldots, \varphi_m(x_1,\cdots,x_n)$ – $[a,b]^n$, $a<b$ to‘plamda aniqlangan uzluksiz funksiyalar sistemasi hamda $u(\cdot)$ – $[a,b]^n$, $a<b$ to‘plamda aniqlangan haqiqiy qiymatli uzluksiz funksiya.

Lemma 1. *H_μ operator chiziqli operator bo‘ladi.*

Isbot.

$(H_\mu(\alpha f+\beta g))(x) = (1-\cos x)(\alpha f + \beta g)(x)$-$\mu \sin x \int_{-\pi}^{\pi} \sin t\big(\alpha f(t) + +\beta g(t)\big)\, dt;$

$\alpha f(x) + \beta g(x) - \cos x\, \alpha f(x) - \cos x \beta g(x) - \mu \sin x \int_{-\pi}^{\pi} \sin t\, \alpha f(t) dt - -\mu \sin x \int_{-\pi}^{\pi} \sin t \beta g(t) dt =$

$$\alpha(1-\cos x) f(x) - \alpha\mu \sin x \int \sin t f(x) dt + +\beta(1-\cos x) g(x) - \beta\mu \sin x \int \sin t g(x) dt = \alpha\big(H_\mu f\big)(x) + \beta\big(H_\mu g\big)(x).$$

Demak, H_μ chiziqli operator ekan.

Lemma isbotlandi.

Lemma 2. *H_μ operator chegaralangan operatordir.*

Isbot.

$$\|H_\mu f\|^2 = \int_{-\pi}^{\pi} \left|(1-\cos x) f(x) - \mu \sin x \int_{-\pi}^{\pi} \sin t f(t) dt\right|^2 dx \le$$

$$\le 2\int_{-\pi}^{\pi}(1-\cos x)^2 |f(x)|^2 dx + 2\mu^2 \int_{-\pi}^{\pi} \left|\sin x \int_{-\pi}^{\pi} \sin t f(t) dt\right|^2 dx \le$$

$$\le 8\int_{-\pi}^{\pi} |f(x)|^2 dx + 2\mu^2 \int_{-\pi}^{\pi} \sin x^2\, dx \cdot \left|\int_{-\pi}^{\pi} \sin t f(t) dt\right|^2 \le$$

$$\le 8\|f\|^2 + 4\mu^2\pi \cdot \int_{-\pi}^{\pi} |\sin t|^2 dt \int_{-\pi}^{\pi} |f(t)|^2 dt = (8 + 8\mu^2\pi^2)\,\|f\|^2;$$

$$\|H_\mu f\| \le 2\sqrt{2(1+\pi^2\mu^2)} \cdot \|f\|.$$

H_μ –chegaralangan operator.

Lemma isbotlandi.

Lemma 3. *H_μ operator o’z-o’ziga qo’shma operatordir.*

$$\int_{-\pi}^{\pi} f(x)\, \overline{(1-\cos x) g(x)} dx - \int_{-\pi}^{\pi} f(x) \overline{\left(\mu \sin x \cdot \int_{-\pi}^{\pi} \sin t g(t) dt\right)} dx =$$

$$= \int_{-\pi}^{\pi} f(x) \overline{\left[(1-\cos x) g(x) - \mu \sin x \int_{-\pi}^{\pi} \sin t\, g(t) dt\right]} d$$

$$= (f, H_\mu, g).$$

H_μ – o’z-o’ziga qo’shma operator ekan.

Lemma isbotlandi.

H operator o‘z-o‘ziga qo‘shmaligi uchun uning spektri haqiqiy sonlardan tuzilgan bo‘ladi.

Ushbu maqolada formula orqali aniqlangan H Fridrixs operatorining spektral xossalarini aniqlovchi algoritm tuzish. Bu yaratilgan algoritmni Maple tizimida
qo‘llab, xossalarni misollarda tekshirib, nazariy ma’lumotlar bilan solishtirishdan iborat:

Operatorni aniqlovchi $\varphi_1(x), \ldots, \varphi_m(x)$ va $u(x)$ funksiyalar ixtiyoriy berilgan qiymatlarida

1. H operatorning muhim spektrini topish;

2. H operatorning diskret spektrini hisoblash, ya'ni muhim spektrdan tashqarida yotgan xos qiymatlarni hisoblash;

3. Natijalarni nazariy ma'lumotlar bilan solishtirishdan iborat.

Tasdiq1. *H operatorni spektri muhim spektr va diskret spektrdan iborat bo'ladi, ya'ni*

$$\sigma(H)=\sigma_{ess}(H)\cup\sigma_{disc}(H),$$

(3)

bunda $\sigma(H)$, $\sigma_{ess}(H)=[m,M]$ va $\sigma_{disc}(H)$ lar mos holda H operatorning spektri, uzluksiz spektri va diskret spektrlari,

$$m=\min_{x\in[a,b]^n}u(x),\quad M=\max_{x\in[a,b]^n}u(x).$$

Tasdiq 2. a) *H operatorning normasi $\|H\|$ uchun ushbu*

$$\|H\|\le\|H_0\|+\|K\|_2$$

(4)

tengsizlik o'rinli, bunda $\|H_0\|=\max\{|m|,|M|\}$,

b) H operatorning spektri $[-\|H\|,\|H\|]$ oraliqda yotadi: $\sigma(H)\subset[-\|H\|,\|H\|]$, shunday qilib, $\sigma(H)\subset[-\|H_0\|-\|K\|_2,\|H_0\|+\|K\|_2]$ munosabat o'rinli bo'ladi.

Tasdiq 3. *Faraz qilaylik K integral yadrosi ajralgan yadroli bo'lsin, ya'ni*

$$K(x_1,\cdots,x_n;s_1,\cdots,s_n)=\sum_{i=1}^{m}\varphi_i(x_1,\cdots,x_n)\varphi_i(s_1,\cdots,s_n)$$

, $m<\infty$ bo'lsin. U holda $L_2([a,b]^n)$ fazoda aniqlangan H operatorning $\sigma_{cont}(H)=[m,M]$ uzluksiz spektrdan tashqaridagi xos qiymatlari to'plami (diskret spektr, karrasi bilan hisoblaganda) m tadan oshmaydi.

Qaralayotgan operatorning diskret spektrini aniqlovchi shartni topamiz. H operatorning xos qiymatiga nisbatan ushbu

$$Hf=zf,\qquad f\ne0,\qquad z\notin[m,M]\qquad(5)$$

tenglamani qaraymiz, yoki

$$u(x)f(x)-\int_a^b\int_a^b\cdots\int_a^b\sum_{i=1}^{m}\varphi_i(x)\varphi_i(s)f(s)ds_1ds_2\cdots ds_n=z$$

,

yoki $\varphi_i(x)$ funksiya integraldan bogliq bo'lmagani uchun oxirgi tenglikni quyidagicha yozamiz

$$u(x)f(x)-zf(x)=\sum_{i=1}^{m}\varphi_i(x)\int_a^b\int_a^b\cdots\int_a^b\sum_{i=1}^{m}\varphi_i(s)f(s)ds_1ds$$

.

Shartga ko'ra $z\notin[m,M]$ bo'lganligi uchun, $u(x)f(x)-zf(x)\ne0$. Bu yerdan oxirgi tenglik quyidagicha bo'ladi

$$f(x)=\frac{1}{u(x)-z}\sum_{i=1}^{m}\varphi_i(x)c_i(f)$$

(6)

bunda

$$c_i=c_i(f)=\int_a^b\int_a^b\cdots\int_a^b\sum_{i=1}^{m}\varphi_i(s)f(s)ds_1ds_2\cdots ds_n$$

. (7)

Endi (2.3) ni (2.4) ga qo'ysak

$$c_i=\sum_{j=1}^{m}c_j\int_a^b\cdots\int_a^b\frac{\varphi_i(s_1,\cdots,s_n)\varphi_j(s_1,\cdots,s_n)}{u(s)-z}ds_1ds_2\cdots ds_n$$

, yoki

$$c_i=\sum_{j=1}^{m}a_{ij}(z)c_j,\quad i=1,2,\dots,n$$

(8)

bunda

$$a_{ij}(z)=\int_a^b\cdots\int_a^b\frac{\varphi_i(s_1,\cdots,s_n)\varphi_j(s_1,\cdots,s_n)}{u(s)-z}ds_1ds_2\cdots ds_n$$

. (9)

Ma'lumki, $\varphi_1(x),...,\varphi_n(x)$ lar berilgan funksiyalar bo'lganligi uchun $a_{ij}(z)$, $i, j=1, 2,..., m$ sonlar har bir $z\notin[m, M]$ larda aniq hisoblashlarni talab etuvchi sonlardir. Agar bu sonlarni hisoblangan deb faraz qilsak, (2.5) tenglikdan c_i, $i=1, 2,..., n$ conlar uchin quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases}(a_{11}(z)-1)c_1+a_{12}(z)c_2+...+a_{1m}(z)c_m=0\\ a_{21}(z)c_1+(a_{22}(z)-1)c_2+...+a_{2m}(z)c_m=0\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ a_{m1}(z)c_1+a_{m2}(z)c_2+...+(a_{mm}(z)-1)c_m=0\end{cases}$$

(10)

Shunday qilib, agar berilgan operator xos qiymatga ega bo'lsa (10) sistema noldan farqli echimga ega bo'lishi kerak ekan. Xuddi shu tarzda mulohaza yritib, teskari tasdiqni ham isbot qilish mumkin, ya'ni (2.7) sistema birorta $z\notin[m, M]$ da noldan farqli echimga ega bo'lsa, shu $z\notin[m, M]$ soni H operator uchun xos qiymat bo'ladi. Algebra kursidan ma'lum bo'lgan quyidagi tasdiqni keltiramiz

Tasdiq 4. (10) tenglamalar sistemasi noldan farqli $\{c_1, \cdots, c_m\}$ yechimga ega bo'lishi uchun $\Delta(z)=0$ bo'lishi zarur va yetarlidir, bunda

$$\Delta(z)=\begin{vmatrix}a_{11}(z)-1 & a_{12}(z) & \dots & a_{1m}(z)\\ a_{21}(z) & a_{22}(z)-1 & \dots & a_{2m}(z)\\ \cdots & \cdots & \cdots & \cdots\\ a_{m1}(z) & a_{m2}(z) & \dots & a_{mm}(z)-1\end{vmatrix}$$

. (11)

Bu Tasdiqdan va yuqoridagi mulohazalarga ko'ra quyidagiga ega bo'lamiz.

Tasdiq 5. *$z\notin[m, M]$ soni H operatorning xos qiymati bo'lishi uchun $\Delta(z)=0$ bo'lishi zarur va yetarlidir.*

Yuqoridagi Tasdiqlarni qo'llab, ushbu xossalarni isbotlash mumkin:

1^0. Faraz qilaylik $n=1$, $u(x_0)=\min\limits_{x\in[a,b]}u(x)$, $K(x,y)=\lambda\varphi(x)\varphi(y)$, $\lambda>0$ va $\varphi(x_0)\neq 0$ bo'lsin. U holda bu parametrlarga mos Fridrixs operatori ixtiyoriy $\lambda>0$ da muhim spektrdan tashqarida yagona xos qiymatga ega bo'ladi.

2^0. Faraz qilaylik $n=1$, $u(x_0)=\min\limits_{x\in[a,b]}u(x)$, $K(x,y)=\lambda\varphi(x)\varphi(y)$, $\lambda>0$ va $\varphi(x)$ analitik funksiya hamda $\varphi(x_0)=0$ bo'lsin. U holda shunday $\lambda_0>0$ son topilib, bu parametrlarga mos Fridrixs operatori ixtiyoriy $\lambda\in[0,\lambda_0]$ da muhim spektrdan tashqarida birorta ham xos qiymatga ega bo'lmaydi.

3^0. (2.1) formula bilan aniqlangan Fridrixs operatorining muhim spektrdan tashqaridagi xos qiymatlari soni m tadan oshmaydi.

Bu yaratilgan algoritmni Maple tizimida qo'llab, 1^0, 2^0, 3^0 xossalarni misollarda tekshirib, nazariy ma'lumotlar bilan solishtirishdan iborat.

Masalani yechish algoritmi

1-qadam. a, b $(a<b)$ sonlar va N natural sonni qiymatlarini kiritish;

2-qadam. $u(x)$ funksiyani kiritish;

3-qadam. $\varphi_1(x), \ldots, \varphi_N(x)$ funksiyalarni kiritish;

4-qadam. $m = \min\limits_{x\in[a,b]^n} u(x)$ va $M = \max\limits_{x\in[a,b]^n} u(x)$ sonlarni hisoblash;

5-qadam. Demak H operatorning muhim spektrini $\sigma_{ess}(H)=[m,M]$ ekan (ekranga chiqarish);

6-qadam. $\| H_0 \| = \max\{|m|,|M|\}$ ni hisoblash;

7-qadam.

$$\| K \|_2 = \sqrt{\int_a^b\int_a^b \cdots \int_a^b | \mathrm{K}(x_1,x_2,\cdots,x_n;s_1,s_2,\cdots,s_n)|^2 \, ds_1 ds_2 \cdots ds_n}$$

sonni hisoblash;

8-qadam. Demak H operatorning diskret spektrini

$[-\| H_0 \| - \| K \|_2, m) \| \cup (\| H_0 \| + \| K \|_2]$

oraliqda yotar ekan (ekranga chiqarish);

9-qadam. $z \notin [m, M]$ sonini kiritamiz;

10-qadam. Ixtiyoriy $i, j \in \{1,\ldots,N\}$ larda

$$a_{ij}(z) = \int_a^b \cdots \int_a^b \frac{\varphi_i(s_1,\cdots,s_n)\varphi_j(s_1,\cdots,s_n)}{u(s)-z} ds_1 ds_2 \cdots ds_n$$

sonlarni hisoblash;

11-qadam.

$$\Delta(z) = \begin{vmatrix} a_{11}(z)-1 & a_{12}(z) & \ldots & a_{1m}(z) \\ a_{21}(z) & a_{22}(z)-1 & \ldots & a_{2m}(z) \\ \ldots\ldots & & & \\ a_{m1}(z) & a_{m2}(z) & \ldots & a_{mm}(z)-1 \end{vmatrix}$$

determinantni hisoblash;

12-qadam. $\Delta = \Delta(z)$ determinantni z o‘zgaruvchining funksiyasi sifatida $[-\| H_0 \| - \| K \|_2, m) \| \cup (M, H_0 \| + \| K \|_2]$ to‘plamdagi nollari mavjudligini aniqlashdan iborat.

13-qadam. Agar $\Delta = \Delta(z)$ determinantni nollari mavjud bo‘lsa bu nollar xos qiymatlarni tashkil etadi aks holda H operatorning diskret spektri yo‘q.

Ishda Maple tizimida bir o‘zgaruvchili Fridrixs modelining muhim spektri va xos qiymatlarini hisoblash algoritmi va dasturiy ta’minoti yaratilgan. Yaratilgan dasturiy ta’minot orqali operatorlarning muhim spektri va xos qiymatlari hisoblab topishda qo’llaniladi.

Misol. Ushbu

$$Hf(x) = [2 - \cos(x) - \cos(x-3)] f(x) - \int_{-\pi}^{\pi} \sum_{n=1}^{2} \cos nx \, c$$

operatorning muhim spektri $\sigma_{ess} = [1.858525596, 2.141474404]$ hamda muhim spektrdan tashqaridagi 2 ta xos qiymatlari, taqriban $z_1 = -1.138927515$ va $z_2 = -1.149035305$ ekanligi ko‘rsatildi.

Maple paketi muxandislik va hisob ishlarini bajarish uchun dasturiy vosita bo‘lib, u professional matematiklar, texnologlar uchun mo‘ljallangan. Uning yordamida o‘zgaruvchi va o‘zgarmas parametrli algebraik va differensial tenglamalarni yechish, funksiyalarni tahlil qilish va ularning ekstremumini izlash, topilgan yechimlarni tahlil qilish uchun jadvallar, grafiklar qurish va boshqa shunga o‘xshash vazifalarni bajarish mumkin. Maple murakkab masalalarni yechish uchun o‘z dasturlash tiliga ham ega.

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