KOSHI-RIMAN SHARTI VA UNING NATIJALARI.

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ABSTRACT

Biz bu maqolada Koshi Riman shartlari va golomorf funksiyalar haqida to`xtalib o`tamiz va Koshi Riman shartining isboti bilan ko`ramiz va funksiyaning analitik yoki difrensiyallanuvchanlikga tekshirib va uni golomorflikga tekshiramiz.

1-teorema. $f(z)$ funksiyaning z_0 nuqtada $f'(z_0)$ hosilasiga ega bo'lish uchun bo'lishi va tengsizliklarning bajarilishi zarur va yetarli.

Isbot. Zarurligi $f(z)$ funksiya z_0 nuqtada $(z_0 \in D)$ $f'(z_0)$ hosilaga ega bo'lsin. Hosila ta'rifiga ko'ra $\lim\limits_{\Delta z \to 0} \frac{\Delta f(z_0)}{\Delta z} = f'(z_0)$,

ya'ni

$$\Delta f(z_0) = f'(z_0)\Delta z + \alpha \cdot \Delta z \qquad (4)$$

bo'ladi bu yerda

$$\Delta z = z - z_0 = (x + iy) - (x_0 + iy) = (x - x_0) + i(y - y_0) = \Delta x + i\Delta y,$$

$$\Delta f(z_0) = f(z) - f(z_0) = [u(x,y) + iv(x,y)] -$$
$$-[u(x_0,y_0) + iv(x_0,y_0)] = [u(x,y) - u(x_0,y_0)] +$$
$$+\mathrm{i}[v(x,y) - v(x_0,y_0)] = \Delta u + i\Delta v$$

bo'lib, α esa Δx va Δy larga bog'liq va ular nolga intilganda nolga intiladi:

$$\lim_{\Delta x \to 0,\ \Delta y \to 0} \alpha = 0$$

Endi $f'(z_0)$ hamda α larni

$$f'(z_0) = a + ib, \quad \alpha = \alpha_1 + i\alpha_2 \qquad \left(\lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \alpha_1 = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \alpha_2 = 0 \right)$$

deb, (4) tenglikni quyidagicha yozamiz:

$$\Delta u + iv = (a + ib)(\Delta x + i\Delta y) + (\alpha_1 + i\alpha_2)(\Delta x + i\Delta y)$$

Bu tenglikdan , haqiqiy xamda mavhum qismlarini tenglab topamiz:

$$\Delta u = a\Delta x - b\Delta y + \alpha_1\Delta x - \alpha_2\Delta y$$
$$\Delta v = b\Delta x + a\Delta y + \alpha_2\Delta x + \alpha_1\Delta y \qquad (5)$$

Demak, $u(x,y)$ va $v(x,y)$ funksiyalar $(x_0\, y_0)$ nuqtada differensiallanuvchi

Ayni paytda $f(z)$ funksiya z_0 nuqtada R^2 ma'noda differensiallanuvchi bo'ladi.

Madomiki, $f(z)$ funksiya z_0 nuqtada , $f'(z_0)$ hosilaga ega ekan,

unda $\Delta z \to 0$, jumladan $\Delta z = \Delta x \to 0$ $(\Delta y = 0)$, $\Delta z = \Delta y \to 0$ $(\Delta x = 0)$ bo'lganda ham

$$\frac{\Delta f(z_0)}{\Delta z}$$

nisbatining limiti har doim $f'(z_0)$ ga teng bo'laveradi, (5) tengliklar

$\Delta z = \Delta x$ $(\Delta y = 0)$ bo'lganda

$$\Delta u = a\Delta x + \alpha_1 \Delta x$$
$$\Delta v = b\Delta x + \alpha_2 \Delta x \qquad (6)$$

$\Delta z = \Delta x$ $(\Delta x = 0)$ bo'lganda esa

$$\Delta u = -b\Delta y - \alpha_2 \Delta y$$
$$\Delta v = a\Delta y + \alpha_1 \Delta y \qquad (7)$$

tengliklarga keladi.

(6) munosabatlardan

$$\frac{\partial u}{\partial x} = a, \qquad \frac{\partial v}{\partial x} = b,$$

(7) munosabatlardan esa

$$\frac{\partial u}{\partial y} = -b, \qquad \frac{\partial v}{\partial y} = a,$$

bo'lishini topamiz.Bu tengliklardan

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

bo'lishi kelib chiqadi.

Yetarliligi. Aytaylik $f(z)$ funksiya z_0 nuqtada R^2 ma'noda differensiallanuvchi bo'lib, teoremada keltirilgan ikkinchi shart bajarilsin. , $u(x,y)$ va $v(x,y)$ funksiyalar (x_0,y_0) nuqtada differensiallanuvchi bo'lgani uchun

$$\Delta u = \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \alpha_1 \Delta x + \alpha_2 \Delta y,$$
$$\Delta v = \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \beta_1 \Delta x + \beta_2 \Delta y.$$

bo'ladi. Bu yerda $\Delta x \to 0$, $\Delta y \to 0$ da $\alpha_1, \alpha_2, \beta_1, \beta_2$ larning har biri nolga intiladi. U holda

$$\Delta f(z_0) = \Delta u + i\Delta v = \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \alpha_1 \Delta x + \alpha_2 \Delta y + i(\Delta v = \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \beta_1 \Delta x + \beta_2 \Delta y)$$

bo'ladi.Teoremaning ikkinchi sharti

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

dan foydalanib topamiz:

$$\Delta f(z_0) = \frac{\partial u}{\partial x}(\Delta x + i\Delta y) - i\frac{\partial v}{\partial y}(\Delta x + i\Delta y) + (\alpha_1 + i\beta_1)\Delta x + (\alpha_2 + i\beta_2)\Delta y = \left(\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}\right)\Delta z + \left[(\alpha_1 + i\beta_1)\frac{\Delta x}{\Delta z} + (\alpha_2 + i\beta_2)\frac{\Delta y}{\Delta z}\right] \cdot \Delta z$$

Bu tenglikda esa

$$\frac{\Delta f(z_0)}{\Delta z} = \frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y} + (\alpha_1 + i\beta_1)\frac{\Delta x}{\Delta z} + (\alpha_2 + i\beta_2)\frac{\Delta y}{\Delta z}, \qquad (8)$$

bo'lishi kelib chiqadi.

Keyingi tenglikdagi

$$(\alpha_1 + i\beta_1)\frac{\Delta x}{\Delta z} + (\alpha_2 + i\beta_2)\frac{\Delta y}{\Delta z}$$

ifoda uchun

$$\left[(\alpha_1 + i\beta_1)\frac{\Delta x}{\Delta z} + (\alpha_2 + i\beta_2)\frac{\Delta y}{\Delta z}\right] \leq [\alpha_1 + i\beta_1] \cdot \left[\frac{\Delta x}{\Delta z}\right] + [\alpha_2 + i\beta_2] \cdot \left[\frac{\Delta y}{\Delta z}\right]$$
$$\leq [\alpha_1 + i\beta_1] + [\alpha_2 + i\beta_2] \leq |\alpha_1| + |\beta_1| + |\alpha_2| + |\beta_2| < \varepsilon$$

bo'ladi, chunki $\Delta z \to 0$ da ya'ni $\Delta x \to 0$ $\Delta y \to 0$ da

$$\alpha_1 \to 0,\quad \beta_1 \to 0,\quad \alpha_2 \to 0,\quad \beta_2 \to 0,$$

Shuni e'tiborga olib , $\Delta z \to 0$ da (8) tenglikda limitga o'tib

$$\lim_{\Delta z \to 0} \frac{\Delta f(z_0)}{\Delta z} = \frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}$$

bo'lishini topamiz. Demak, $f(z)$ funksiya z_0 nuqtada $f'(z_0)$ hosilaga ega va

$$f'(z_0) = \frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}$$

bo'ladi. Teorema isbot bo'ldi.

Teoremada keltirilgan (3) shartlar Koshi-Riman shartlari deyiladi.

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