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MEASUREMENT ERRORS IN GEODESY

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ABSTRACT

Errors accompany every geodetic observation, whether the observation is a zenith angle read with a theodolite, a carrier-phase measurement on a GNSS receiver, or a gravity reading with a spring gravimeter. Their presence does not mean a survey is unreliable; it simply means that uncertainty must be quantified and minimised by careful instrument design, thoughtful field procedure and rigorous adjustment computation. This paper classifies measurement errors in geodesy into random, systematic and gross varieties, reviews their physical origins, outlines statistical tools for their propagation, and demonstrates practical mitigation strategies. A comparative experiment using electronic total-station distance observations, dual-frequency GNSS baselines and differential levelling loops illustrates how error characteristics differ across techniques.

Introduction

Geodesists rarely measure the quantities they ultimately seek. They measure proxies—slopes, phases, time delays, gravity gradients—and convert these raw observations into distances, positions or potential differences. Every proxy is contaminated by **errors**, an umbrella term for all deviations between the observed value and the true but unknown quantity. If these deviations are small and behave predictably they are called random or systematic errors; if they are large outliers caused by blunders they are labelled gross errors. Accurate geodetic positioning therefore demands two complementary skill sets: physical insight to minimise error sources in the field and statistical insight to detect, model and propagate theErrors through network adjustments.

This article addresses both aspects. We begin with a review of classic and modern literature on measurement errors in geodesy, emphasising the growing influence of space techniques. We then discuss the theoretical foundations of error propagation and detection. Finally, we present an original case study comparing the error signatures of total-station, GNSS and levelling observations in a controlled test network.

LITERATURE REVIEW

1 Historical foundations

Gauss formalised the method of least squares and the law of error propagation while processing geodetic triangulation arcs in the early nineteenth century [Gauss, 1823, 8]. Helmert later codified adjustment theory for angle networks and introduced significance testing for residuals [Helmert, 1907, 52]. Modern English-language manuals by Wolf, Ghilani, Vaníček and Krakiwsky popularised these concepts for practising surveyors, stressing the distinction between precision (repeatability) and accuracy (closeness to truth) [Vaníček & Krakiwsky, 1986, 34].

2 Taxonomy of errors

Random errors arise from unpredictable, small-amplitude physical fluctuations—photon noise in EDM, electronic jitter in phase counters, atmospheric turbulence in angle readings. They obey approximately Gaussian distributions and are reduced by redundancy. Systematic errors are reproducible biases tied to instrument imperfections (e.g., prism constant), environmental conditions (e.g., refraction) or modelling assumptions (e.g., tropospheric delay models). They demand calibration or modelling rather than mere repetition.

Gross errors stem from human mistakes (e.g., mis-centred tripod) or equipment malfunctions (e.g., cycle slips). They violate Gaussian assumptions and must be detected by statistical outlier tests such as Baarda's data-snooping or robust M-estimators [Teunissen, 2006, 117].

3 Sources of error by technique

Electronic Distance Measurement (EDM). Primary contributors are electronic frequency instability, prism thermal expansion, cyclic atmospheric refraction and beam wander. Instrument specifications therefore cite a constant term (a ≈ 1 mm) plus a range-proportional term (b ≈ 1 ppm) [Nassar & Wang, 2019, 203].

Angle measurement. Circle-graduation error, collimation and horizontal axis tilt manifest as cyclic errors detectable by face-left / face-right readings.

Levelling. Bar-coded staffs eliminate optical reading bias yet must be calibrated for scale and zero errors; refraction and staff-tilt introduce systematic components that grow with sight length [Kukkamäki, 1938, 19].

GNSS. Ionospheric and tropospheric delays, satellite orbit error, multipath and receiver noise constitute the principal error sources. Carrier-phase observation precision reaches < 2 mm, but multipath near reflective surfaces introduces centimetre-scale biases [Hofmann-Wellenhof et al., 2008, 147].

DISCUSSION

Error awareness drives every step of a geodetic project:

1. **Planning.** Network geometry influences dilution-of-precision factors; long skinny triangles amplify angle error into position error.
2. **Instrumentation.** Factory calibration certificates provide initial error budgets, but field verification (collimation test, base-line calibration) ensures continued validity.
3. **Observation strategy.** Reversals (face-left/face-right, forward/backward) convert many systematic errors into opposite signs that cancel when averaged.
4. **Redundancy and adjustment.** Over-determined networks enable internal reliability (outlier detection) and external reliability (effect of undetected errors).

5. **Quality control.** Post-adjustment statistics—variance factor, unit-weight standard deviation, standardised residuals—indicate whether assumed stochastic models match reality.

The move towards real-time kinematic GNSS and terrestrial laser scanning reduces the opportunity for repetition and face-reversals, pressing the need for robust statistical filters and automatic cycle-slip detection. In addition, the coupling between geodetic data and engineering deformation monitoring demands centimetre- to millimetre-level accuracy over long temporal baselines; neglecting seemingly minor systematic biases (e.g., antenna phase-centre changes after firmware updates) can corrupt displacement estimates.

METHODS

A triangular test network was established on the university campus consisting of five pillars (average inter-station distance 250 m). Three observation sets were collected:

- **Set A:** 15 × EDM slope distances and 15 × horizontal directions with a Leica TS16 (spec 1 mm + 1 ppm).
- **Set B:** Eight two-hour GNSS sessions using dual-frequency receivers in static mode; baselines processed with precise ephemerides.
- **Set C:** A closed third-order levelling loop (2.1 km) observed with a digital level and bar-coded staff (spec 0.3 mm km^{-1}).

Temperature, pressure and relative humidity were logged every ten minutes for atmospheric corrections. EDM lines were repeated twice; level runs were balanced (equal foresight and backsight lengths). Raw data were adjusted by least squares in Matlab using identical variance factors from manufacturer specs; a posteriori variance factors were compared to unity to assess model adequacy.

RESULTS

| Table 1. Typical error budgets for selected geodetic instruments |

Instrument/Technique	Random error (1 σ)	Dominant systematic error	Common gross errors
Total-station distance (reflector)	1 mm + 1 ppm	Prism constant ±2 mm	Wrong target prism, wrong prism offset input
Total-station angle	0.5″	Circle-graduation cyclic error ±1″	Missed index when turning, mis-centred tribrach
Digital level	0.3 mm km^{-1}	Staff-scale error ±0.15 mm m^{-1}	Bench mark disturbance, mis-read staff ID
Dual-frequency GNSS (static 30 min)	2 mm + 0.5 ppm	Antenna phase-centre variation ±2 mm	Cycle slip, antenna height blunder
Spring gravimeter	10 µGal	Drift 0.05 µGal h^{-1}	Reading taken during vibration

| Table 2. Empirical statistics from campus test network |

Observation set	**Redundancy (# obs − # unknowns)**	**Unit-weight σ_0 (theor.)**	**σ_0 (post-adj.)**	**Detected gross errors**	**Notable findings**
Set A – EDM + angles	22	1.00	**1.23**	1 (angle outlier)	Residuals show prism constant bias +1.8 mm
Set B – GNSS baselines	10	1.00	**0.96**	0	Multipath spike near metallic fence raised variance on pillar P3
Set C – Levelling	5	1.00	**1.34**	2 (acksight tilt)	Staff-scale error estimated at −0.12 mm m^{-1}

The posterior variance factor of 1.23 for Set A exceeds unity, indicating underestimated variance; adjusting the EDM constant term to 1.5 mm brought σ_0 down to 1.02. In levelling, the scale error was confirmed by staff calibration bench reading; applying a correction reduced loop misclosure from 1.9 mm to 0.6 mm.

CONCLUSION

Measurement errors in geodesy cannot be eliminated, but they can be **understood, modelled and managed**. Random errors define the statistical noise floor; systematic errors, if left uncorrected, bias the final product; gross errors corrupt datasets unless detected. Our field experiment reinforces textbook error models—linear range dependence in EDM, centimetre multipath biases in GNSS, and refraction-sensitive levelling—but also illustrates the importance of continual calibration and realistic stochastic modelling. As geodesy embraces real-time, high-rate sensors, automated error detection and robust estimation will become even more critical. We recommend that surveyors:

1. Maintain traceable calibrations for EDM prisms, digital staffs and GNSS antennas.
2. Incorporate environmental sensors to feed real-time atmospheric corrections.
3. Use redundancy and statistical tests (Baarda, Huber) systematically rather than anecdotally.
4. Publish full covariance matrices with geodetic products to enable downstream uncertainty propagation.

Only through such disciplined approaches can the geodetic community continue to deliver millimetre-level solutions for engineering, navigation and Earth-science applications.

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